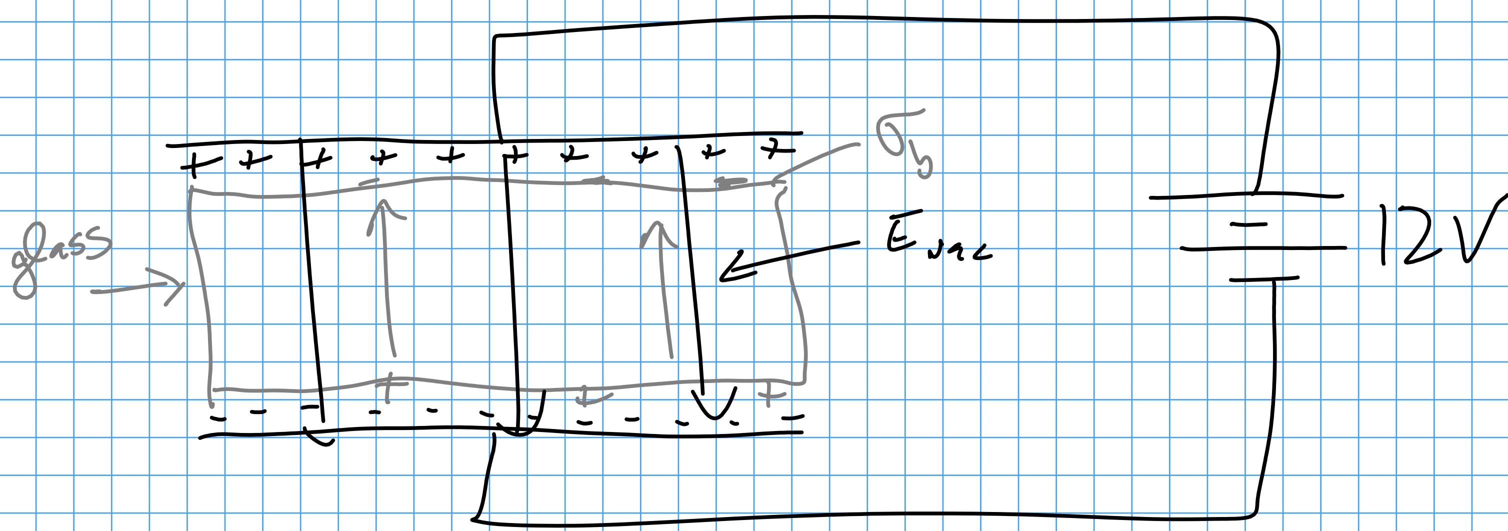


Need to know $\sigma_b = \vec{P} \cdot n$

assume linear in $\vec{E}$

Assume: $\vec{P}$ is due to induced dipole $\left(\vec{p} = \alpha \vec{E}\right)$

$\frac{\text{dipole moment}}{\text{vol}}$

little p $\frac{\text{dipole mom}}{\text{atom}}$

$\vec{P} \propto \vec{E}$

$\vec{P} = \epsilon_0 \chi_e \vec{E}$

e is for electric

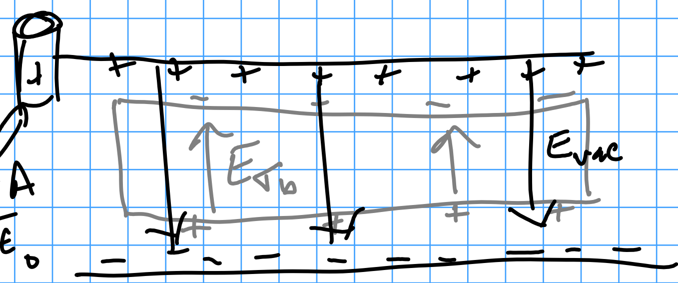
If we know $\frac{\#\text{ atoms}}{\text{vol}}$ then α related to $\epsilon_0 \chi_e$

Linear material $\vec{P} = \epsilon_0 \chi_e \vec{E}$

χ_e ← electric (later B for magnetic)

Assumptions
- No permanent dipole moments
- No non-linear terms

$EA = \frac{\sigma A}{\epsilon_0}$



E_{vac} in cap $= \frac{\sigma_f}{\epsilon_0}$ ← free

$$E_{vac} - E_{glass} = E_{tot}$$

$$\frac{\sigma_f}{\epsilon_0} - \frac{\sigma_b}{\epsilon_0} = E_{tot}$$

(σ_b ← bound)

$$\underline{\sigma_b} = \vec{P}\cdot\hat{n} = \underline{\epsilon_0 \chi_e E_{tot}}$$

$$\frac{\sigma_f}{\epsilon_0} - \frac{\epsilon_0 \chi_e E_{tot}}{\epsilon_0} = E_{tot}$$

$$\frac{\sigma_f}{\epsilon_0} = E_{tot} + \chi_e E_{tot} = E_{tot}(1+\chi_e)$$

$$E_{tot} = \frac{\sigma_f}{\epsilon_0} \frac{1}{(1+\chi_e)} = \frac{E_{vac}}{K}$$

K ← dielectric const

$$\sigma_b = \epsilon_0 \chi_e E_{tot} = \cancel{\epsilon_0}\chi_e \frac{\sigma_f}{\cancel{\epsilon_0}} \frac{1}{(1+\chi_e)} = \sigma_f \frac{\chi_e}{(1+\chi_e)}$$

A "simpler" way

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} = \frac{\rho_{free}+\rho_{bound}}{\epsilon_0} = \frac{\rho_f+\rho_b}{\epsilon_0}$$

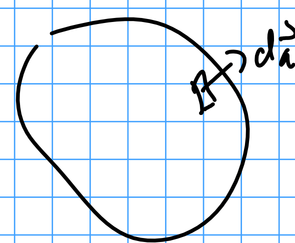
$$\rho_b = -\vec{\nabla}\cdot\vec{P}$$

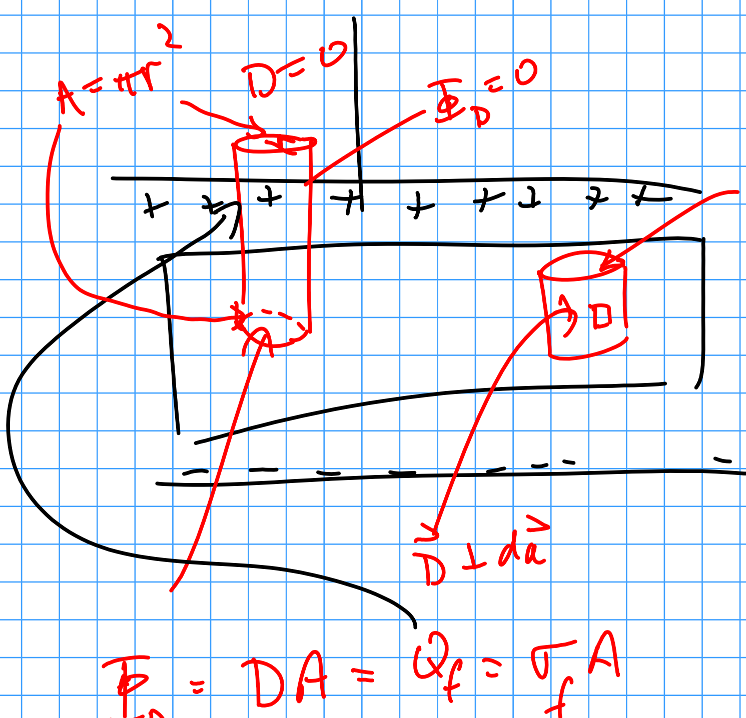
$$\vec{\nabla}\cdot\epsilon_0\vec{E} = \rho_f - \vec{\nabla}\cdot\vec{P} \Rightarrow \vec{\nabla}\cdot(\underbrace{\epsilon_0\vec{E}+\vec{P}}_{\equiv \vec{D}}) = \rho_f$$

$$\vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\int \vec{\nabla}\cdot\vec{E}\, d\tau = \oint \vec{E}\cdot d\vec{a}$$

$$\underbrace{\int \vec{\nabla}\cdot\vec{D}\, d\tau}_{=\rho_b} = Q_f = \oint \vec{D}\cdot d\vec{A}$$





$$\oint \vec{D}\cdot d\vec{a} = \frac{Q_f}{\epsilon_0}$$

$\uparrow \vec{D}$ $\uparrow$ direction

$$\Phi_D = DA = Q_f = \sigma_f A \qquad D = \sigma_f$$

Note in vacuum Gaussian surface does not enclose charge

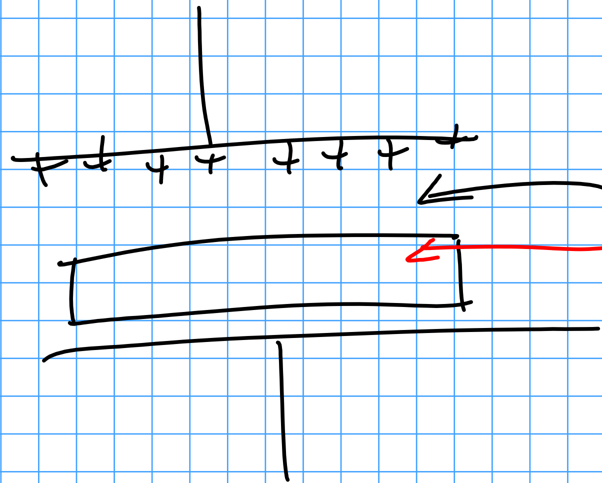
$$\oint \vec{E}\cdot da = \frac{Q_{encl}}{\epsilon_0} = \frac{\phi}{\epsilon_0}$$

$$\Phi^{top}_{cap} + \Phi^{bottom}_{cap} = 0$$

$$\Phi^{top}_{cap} = -\Phi^{bottom}_{cap}$$

$$E^{top}\cancel{A} = E^{bottom}A$$

This doesn't help find E.



$$D = \sigma_f = \epsilon_0 E + \overset{0}{\overset{\shortparallel}{P}} \Rightarrow E = \frac{\sigma_f}{\epsilon_0}$$

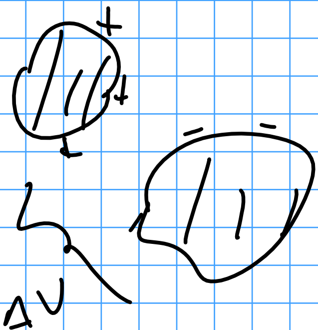
$$D = \sigma_f = \epsilon_0 E + P = \epsilon_0 E + \epsilon_0 \chi_e E$$

$$\sigma_f = \epsilon_0 E (1 + \chi_e)$$

$$E^{tot}_{glass} = \frac{\sigma_f}{\epsilon_0} \frac{1}{(1+\chi_e)}$$

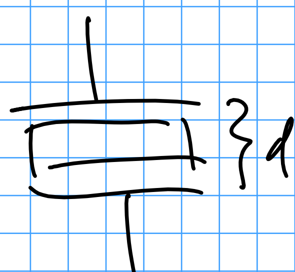
Capacitance

$$C = \frac{Q}{|\Delta V|} \quad ? \qquad Q_{free} = \sigma_f A$$



$$\Delta V = -\int \vec{E} \cdot d\vec{l} \rightarrow E_{tot}\, d$$

E_{tot}



$$C = \underbrace{\frac{\epsilon_0 A}{d}}_{= C_{vac}} (1 + \chi_e)$$