Nonlinear Optics
Homework 1
due Tuesday, 23 Jan 2007

- **Problem 1: Boyd 1.1 Consult Appendix B for help.**

  - Note, the value of $\chi^{(2)}{}_{yyy} = 1.3 \times 10^{-7}$esu (not a positive 7 as in the book). Also note equation B.5 has the wrong exponent.
  - In the literature, SI values of these coefficients are quoted in terms of picometers/V (pm/V).
  - As part of this problem, use the equivalence of the expressions for energy density in the two systems of units ($\epsilon 0\, E^2$ in SI, $E^2 / 4\,\pi$ in Gaussian) to derive equation B.10.

- **Problem 2: Boyd 1.4**
  - The first part comes directly from the equation.
  - For the explanation, consider the nature of the nonlinear restoring force.

- **Problem 3: Boyd 1.5**
  - you may do this problem for third harmonic generation (all input ω's the same) rather than for the general case.
  - to estimate the order of magnitude of the contributions, use the scaling arguments discussed in the reading. Assume there are no resonances for this part.

- **Problem 4:**
  The commonly-used nonlinear crystal BBO (beta- barium borate) is a uniaxial crystal with nonlinear coefficients d31 = 0.16 pm/V and d22 = 2.2 pm/V. The effective nonlinear coefficients as a function of the direction of the fundamental beam for this crystal are given in equation 1.5.30. The angle θ is fixed according to phase matching, but we are free to choose whatever value of ϕ that will optimize the magnitude of the nonlinearity $|d_{eff}|$.

  For an input wavelength of 800nm, the type I phase matching angle is θ = 29 deg and for type II, the angle is θ = 42.3 deg.
  - Plot d_{eff} vs ϕ for both cases, and determine the optimum value.
  - Which phase matching type gives a higher nonlinearity?

- **Problem 5:**
  Use NDSolve[ ] in *Mathematica* (or another differential equation solver if you're using another program) to solve for the displacement of the classical nonlinear oscillator (equation 1.4.1). In *Mathematica*, use the help on NDSolve for examples of how to program it. It is actually quite easy:

```
NDSolve[{x''[t] + 2 γ x'[t] + ω0^2 x[t] + a x[t]^2 == - Cos[ω t],
   x[0] == 0, x'[0] == 0}, x, {t, 0, tMax}];
```

Use $\omega 0 = 3$; $\omega = 1$; $\gamma = 0.1$;
a. Make a plot of x(t) for a = 0, and for some value of a that shows some different output. Explain the origin of the transients you see at early times.

b. Subtract the linear output function from the nonlinear function, and plot it. Increase the value of the nonlinear parameter a that until you see departures from what you would expect from the perturbation theory shown in the book. Discuss what you find.