Hmwk soln assign 7

1.) See second page of Feb 28 lecture for E inside the region of B.

inside

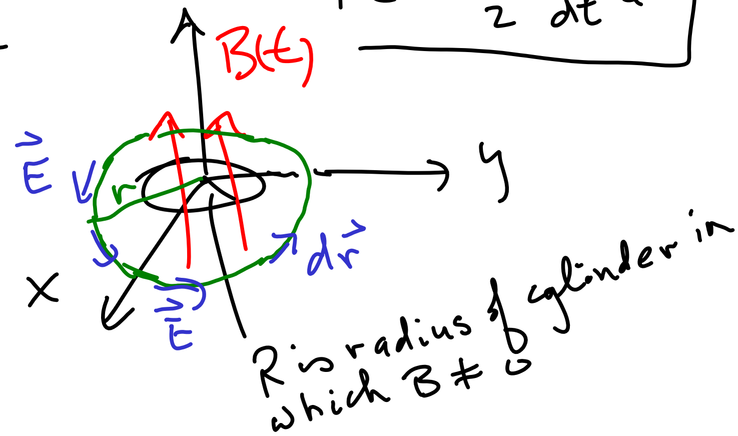
$$\vec{E} = -\frac{r}{2}\frac{dB}{dt}\hat{\varphi}$$

Outside the region of B we have

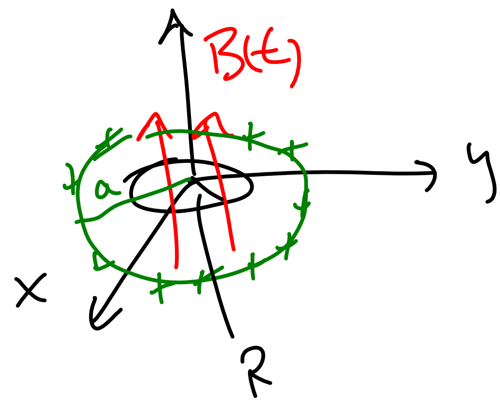
$$\oint \vec{E}\cdot d\vec{r} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a} = -\frac{dB}{dt}\pi R^2$$

$$E\, 2\pi r = -\frac{dB}{dt}\pi R^2$$

$$\boxed{\vec{E} = -\frac{R^2}{2r}\frac{dB}{dt}\hat{\varphi} \text{ outside}}$$




2.) From the Feb 28 lecture the Faraday electric field is tangential and given by




The force on a charge on the rim dq is

$$d\vec{F} = dq\,\vec{E} = -\lambda r\, d\theta\, \frac{R^2}{2r}\frac{dB}{dt}\hat{\varphi}$$

The torque generated is

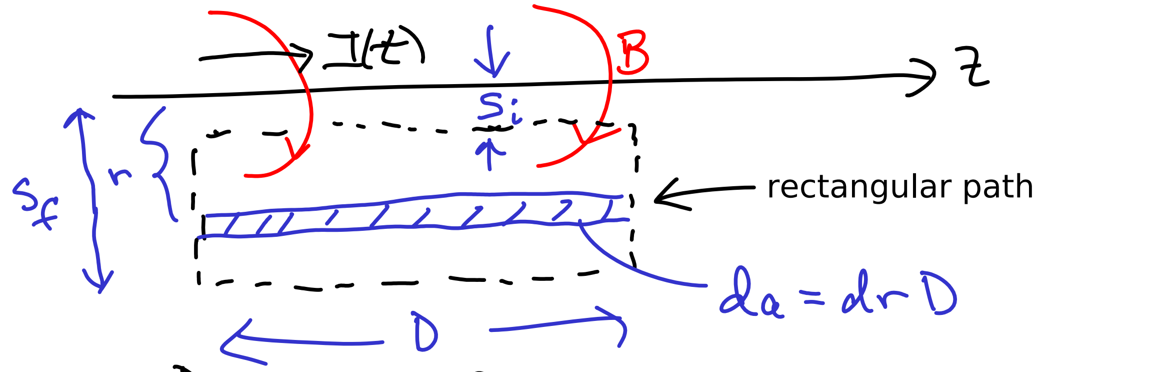
$$d\vec{\tau} = \vec{r}\times d\vec{F} = -\lambda r\, d\theta\, \frac{R^2}{2}\frac{dB}{dt}\hat{z}$$

$$\tau = \int d\tau = -\frac{\lambda r R^2}{2}\frac{dB}{dt}\int_0^{2\pi} d\theta = \frac{dL}{dt}$$ Multiply both sides by dt

$$-\pi\lambda r R^2\int_{B_0}^{0} dB = \int_{L_i=0}^{L_f} dL \Rightarrow \pi\lambda r R^2 B_0 = L_f$$

How can angular momentum be conserved if the ring had none and now has some?

3.)




$$\oint \vec{E}\cdot d\vec{r} = \int_0^D |\vec{E}|\underbrace{|d\vec{r}|}_{dz}\cos 0 + \int_D^0 |\vec{E}|\underbrace{|d\vec{r}|}_{dz}\cos 0 = E(s_i)\,D - E(s_f)\,D$$

$$\Phi_m = \int \vec{B}\cdot d\vec{a} = \int |\vec{B}||d\vec{a}|\cos 0 = \int_{s_i}^{s_f} \frac{\mu_0 I(t)}{2\pi r}\, dr\, D$$

$$= \frac{\mu_0 I(t)\, D}{2\pi} \ln\left(\frac{s_f}{s_i}\right)$$

$$\oint \vec{E}\cdot d\vec{r} = -\frac{d\Phi}{dt} \Rightarrow \quad \vec{E}(s_f) = \frac{\mu_0}{2\pi}\frac{dI}{dt}\left(\ln s - \text{const}\right)\hat{z}$$

4.) Find L from its defn

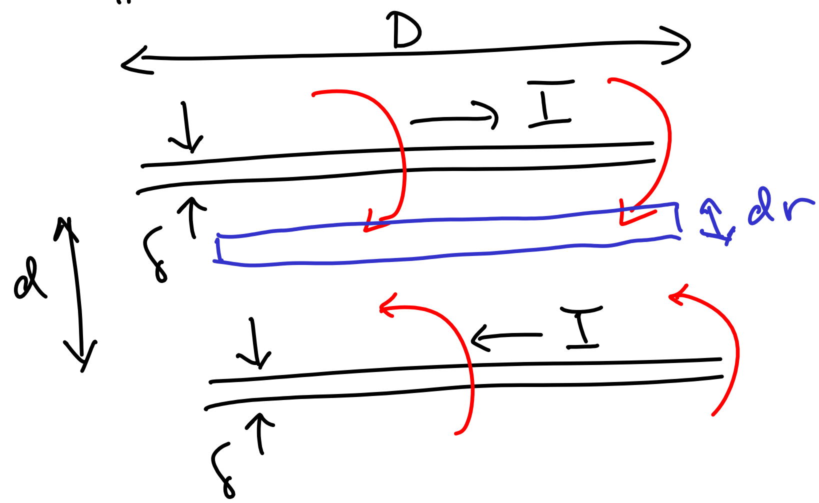
$$\Phi_m \equiv L I$$

Calculate the flux between two very long wires of diameter $\delta$

Assume there is no contribution to this flux from within the wire. Calculate the flux for one wire out to the second and double this flux.

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\Phi = 2\,\frac{\mu_0 I}{2\pi}\int_{\delta/2}^{d-\delta/2} \frac{dr}{r} = \frac{\mu_0 I D}{\pi}\ln\left(\frac{d-\delta/2}{\delta/2}\right)$$




The size of the wire is important in determing the self inductance.

5.)

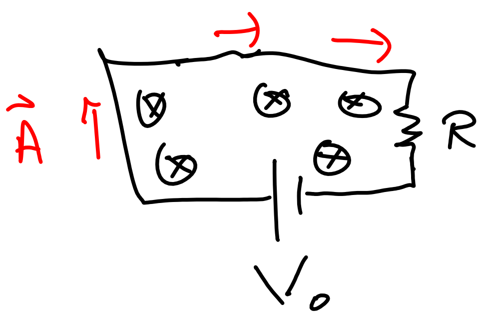
$$\Phi_m = \int \vec{B}\cdot d\vec{a} = \int \vec{A}\cdot d\vec{r} = LI$$

$$\int \vec{B}\cdot d\vec{a} = \int \vec{\nabla}\times\vec{A}\cdot d\vec{a} = \int \vec{A}\cdot d\vec{r}$$

Show using Stokes theorem

why? Defn $\Phi_m = LI$

What does the line integral mean for this circuit? $\int \vec{A}\cdot d\vec{r}$




$\oint \vec{A}\cdot d\vec{r}$

|| from above

Using $W = \frac{1}{2}LI^2 = \frac{1}{2} I\, LI$

show

$$W = \frac{1}{2} I \oint \vec{A}\cdot d\vec{r}$$

bring inside

$$W = \frac{1}{2} \oint \vec{A}\cdot \vec{I}\, dr$$ where $I$ points along $d\vec{r}$

Generalize this result to current density throughout a volume

$$W = \frac{1}{2}\int \vec{A}\cdot\vec{J}\,d\tau$$

Let's write this entirely in terms of the magnetic field using $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

$$W = \frac{1}{2\mu_0}\int \vec{A}\cdot\vec{\nabla}\times\vec{B}\,d\tau$$

Integration by parts moves the curl from B to A using this identitiy

$$\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})$$

$$\vec{A}\cdot\vec{\nabla}\times\vec{B} = \vec{B}\cdot\vec{\nabla}\times\vec{A} - \vec{\nabla}\cdot(\vec{A}\times\vec{B})$$

$$W = \frac{1}{2\mu_0}\left[\int B^2\,d\tau - \int \vec{\nabla}\cdot(\vec{A}\times\vec{B})\,d\tau\right]$$

$\downarrow$ divergence

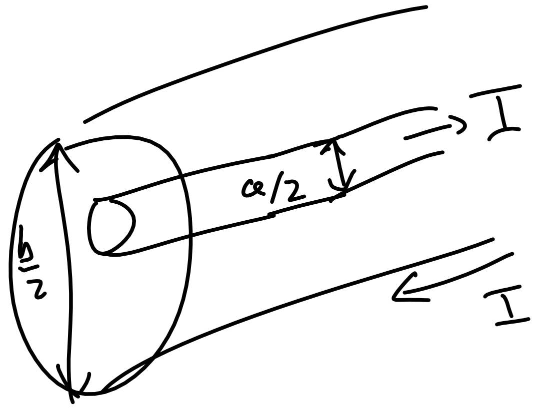
$$\int \vec{A}\times\vec{B}\cdot da$$

Apply this identity and the divergence theorem to show that

$$W = \frac{1}{2\mu_0}\left[\int_{\text{vol}} B^2\,d\tau - \oint_{\text{surface}} \vec{A}\times\vec{B}\cdot da\right]$$

Far from the currents the integrand of the second term goes to zero leaving the energy expressed only in terms of B.

6.)

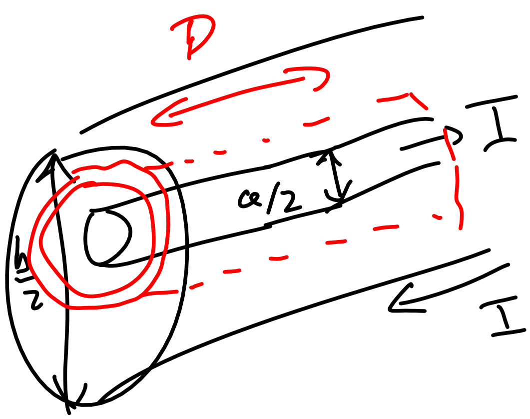



Ampere's law gives $B = \frac{\mu_0 I}{2\pi r}$ $\hat{\phi}$ inside & 0 outside

Energy per unit volume of the coaxial cable is

$$\frac{1}{2\mu_0} B^2 = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2$$

Energy in the red shell of length D and thickness dr is




$$\frac{B^2}{2\mu_0} 2\pi r\, dr\, D$$

$$= \frac{\mu_0 I^2}{8\pi^2 r^2} 2\pi D r\, dr$$

The total energy in length D is

$$\int_a^b \frac{\mu_0 I^2 D}{4\pi} \frac{dr}{r} = \frac{\mu_0 I^2 D}{4\pi} \ln\frac{b}{a} = \frac{1}{2} L I^2$$