PHGN 462 Homework 8

1) Pollack and Stump 12.18

2) Pollack and Stump 12.20. You may have seen something like this already, but just in case you haven't, I would be negligent if I let you out of here without examining the cosmological redshift. If you have no idea what cosmological redshift is, please read at least part of

http://en.wikipedia.org/wiki/Cosmological_redshift

before you do the problem.

3) Pollack and Stump 12.25, part a only. I claimed in class that we want to write things as four vectors whenever possible because they follow Lorentz transforms. The usefulness of that probably isn't going to sink in very far unless you look at how things that aren't four vectors transform. So here you get to work out how classical force transforms, and see for yourself how it's much less pretty than Minkowski force (which you transform by just slapping it with a Lorentz matrix).

4) I am known for my generosity. Since no one really had time to properly address the third part of the most recent exam, I'll give you another go at it, plus a little relativistic bonus!

a) We solved for the Lienard-Wiechert potentials and got expressions for the general fields made by a moving point charge. If the charges are moving in a straight line (ie, not accelerating), the expressions reduce down to:

$$\vec{E}(\vec{x},t) = \frac{q}{4\pi\varepsilon_0} \frac{1-{v^2}/{c^2}}{\left(1-\frac{v^2}{c^2}sin^2(\theta)\right)^{3/2}} \frac{\hat{R}}{R^2} \qquad \vec{B}(\vec{x},t) = \frac{\vec{v}\times\vec{E}}{c^2}$$

Where $\vec{R}$ represents $\vec{x}-\vec{x}'$ evaluated at the retarded time, and θ is the angle between $\vec{v}$ and $\vec{R}$.

Let's not forget that currents are nothing more than moving charges. We can represent a steady current by a 1-D line of charges all moving in the same direction. So consider a linear charge of density λ moving with some speed v. Use the given field expressions to find the fields E & B some distance d from the line of charges. You may assume that the charges have been in place and moving at the same speed in the same direction *forever*.

Various hints so you don't get totally stuck: In 1-D, $dq = \lambda dl$, and $I = \lambda v$. Let the line lay on the z-axis, with the charges moving in the $+\hat{z}$ direction. Draw a picture and use some trig to re-express R, $\hat{R}$, z, and dz in terms of θ, d and $d\theta$. Then you can use the substitution $u = \cos\theta$ to find E. B follows quickly.

b) That was kind of painful. Certainly more difficult than just using Gauss's law and Ampere's law to find the fields. But it's pretty cool that we can get these results just from the general fields of a point charge. You know what's even more fun? Consider a linear charge of density λ at rest. What kind of E &B fields does it make? Now use relativistic transformation rules to get from the rest frame to some other frame moving with speed v along the length of the line of charge. What are the fields in the moving frame? Use transformation matrices if you can, but you don't have to.

c) The answers to a & b aren't exactly the same. They're close, but they differ by a factor of gamma. I've gone pretty far out of my way to say that special relativity is baked into E&M, so why did we get a difference here?