Homework 11 due 5 Dec. 2007 5 pm
PH462 EM Waves and Optical Physics posted: 25 Nov. 2007

1) When we use a spectrometer, our signal (the spectrum, or more precisely, the spectral intensity) is proportional to $\left|E_{in}(\omega)\right|^2$, where $E_{in}(\omega) = \Im\left\{E_{in}(t)\right\}$. Consider two identical pulses of light entering a spectrometer with a time delay τ. Show that when the spectral intensity is measured, we see interference fringes in the spectrum. (This is the starting point for the field of spectral interferometry.)
   a. Calculate the measured spectrum in the case where the pulses have a rectangular shape (see the definition of the rect function on the transform sheet:
   $$E_{in}(t) = E_0 rect\left(\frac{t}{t_0}\right)e^{-i\omega_0 t} + E_0 rect\left(\frac{t-\tau}{t_0}\right)e^{-i\omega_0(t-\tau)}$$
   b. Make a plot of the spectrum for two different time delays, showing the variation in the fringe spacing with delay. Determine how to calculate the time delay by measuring the spectral separation of the fringes.
   c. When a beam passes through the entrance slit of an imaging spectrometer, the output plane is a two-dimensional image. Each row will give the spectrum that corresponds to a spatial position along the entrance slit. With this arrangement, we can use spectral interferometry to measure the spatial wavefront differences between a reference pulse and the delayed pulse. Suppose the reference pulse has a flat wavefront (constant phase across the beam). Let the delayed pulse have a diverging wavefront $E_0 rect\left(\frac{t-\tau}{t_0}\right)e^{-i\omega_0(t-\tau)}e^{i\phi(y)}$ (where you use the paraxial approximation from homework 2 to calculate $\phi(y)$). Make a density plot of what one would observe at the output of the imaging spectrometer. Choose your parameters to make the effect clearly visible. Either describe or make another image of what the output would look like if the diverging beam had chromatic aberration (effective radius of curvature of the wavefront would depend on the wavelength).

2) Find the Fourier transform of a decaying series of exponentials: $f(t) = \sum_{n=0}^{\infty}\delta(t - nt_0)\exp[-\alpha n]$.

   How can this result be used to understand the Fabry-Perot interferometer? Describe how t_0 and α relate to the physical parameters of the Fabry-Perot.

3) A beam of white light is directed at normal incidence on a transmission diffraction grating, and the diffraction orders -2, -1, 0, 1, 2 are observed on a screen. Calculate the diffraction angles of 400nm and 700nm for these orders and use them to sketch the visible portion of what you would observe on the screen. Estimate the groove density of the grating (lines/mm) with the knowledge that we can just see the full range of wavelengths in the +/- 2 order.

4) HM 11-16. Use the fact that T = 1-R-A to convert the standard Fabry-Perot expression.

5) HM 11-18

6) HM 11-19

7) HM 12-7. The result of this calculation is that the Fraunhofer diffraction pattern, which is normally found at long distances from the diffracting aperture, can be viewed at the focal plane of a lens. Since the Fraunhofer pattern is essentially a 2D Fourier transform, a lens takes the Fourier transform of what field is at its input.

8) A collimated laser beam with the following electric field profile is incident on a lens with focal length *f*: $E_{in}(r) = E_0 \exp[-r^2 / w^2]$. Use Cartesian coordinates to calculate the Fraunhofer diffraction pattern at the focal plane of the lens. Express the new spot radius as a function of the wavelength, *f* and *w*. The "radius" at the input and output is characterized by the radial distance to the 1/*e* point of the field. You may neglect the truncation of the input field by the edges of the lens.

9) A beam is propagating inside a high-order mode of a rectangular waveguide with the field profile:
$$E(x,y) = E_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) rect(x/a)\, rect(y/b)$$
It leaves the waveguide and propagates in free space. Calculate the far-field intensity pattern using the Fraunhofer/Fourier method. Compare the angles of the maxima of the pattern to the internal ray angles for the mode inside the waveguide.