PHGN 480: Laser Physics due 7 Oct 2011
Homework 5

1) Griffiths problem 9.14. For part (b) see how far you can get without actually doing any integrals: first use the selection rules as described in section 9.3.3 to determine which matrix elements are zero; then, as part of calculating the ratios see how much cancels before doing the integration. In the end (for part c), you'll have to do the integration, and when you do, you can use Mathematica. Note the spherical harmonics are built-in. As part (d), calculate the natural linewidth for the transition out of the |300> state, and use your relation from problem 2 above to calculate the stimulated emission and absorption cross-sections.

2) In the last few years there is a lot of interest in diode pumped alkali vapor lasers. The idea is that while diode lasers are very efficient, they don't have very good beam quality. So if you can efficiently pump another laser, then you can focus the output beam much more tightly. NIST has some databases that are very useful for looking up energy levels and transition rates.
   a. Go to the NIST atomic energy levels database: http://physics.nist.gov/PhysRefData/ASD/levels_form.html, enter Rb I and press "Retrieve Data". This will bring you to a page that lists the energy levels and the spectroscopic terms for the excited states of the rubidium atom. (The Rb^+ ion would be called Rb II.) Find the two energy levels for the 5p configurations (italics on the energy level and the "o" superscript on the term mean that state has odd parity). By the way, this doublet is the equivalent to the sodium D-line doublet.
      i. Make a schematic (not to exact scale) 3-level energy level diagram, listing the S, L and J values for each level.
      ii. Explain the reason why each of the three levels has those values of S, L and J.
      iii. Calculate the wavelengths needed to pump to the higher level from the ground state (λ_{31}) and for the lasing transition back (λ_{21}).
   b. Now go to the atomic spectra database: http://physics.nist.gov/PhysRefData/ASD/lines_form.html
   Put in Rb I for the spectrum, and choose a wavelength range of +/- 20nm around the wavelengths you found in part a. This chart shows more information, including spontaneous emission rates A_{kl}, and the degeneracy factors (g's).
      i. Write down the values for A_{31} and A_{21}.
      ii. How do the degeneracy factors relate to the spectroscopic state of the electron?
   c. Helium is introduced as a buffer gas, to collisionally broaden the Rb lines and to collisionally transfer population from the ${}^2P_{1/2}$ and ${}^2P_{3/2}$ levels. Suppose this relaxation time from 3 to 2 is τ_c. Supposing that there is no beam at λ_{21}, but the pump beam is on. Write the rate equations for each of the three levels.

For compactness, you can write $W_{13}N_1 / g_1$ as the absorption rate/vol and $W_{13}N_3 / g_3$ as the stimulated emission rate/vol.

d. As a 3-level system, it is difficult, but not impossible to get CW gain. Set each of the time derivatives for your rate equations in the previous part, and use the equation for dN_3 / dt to solve for N_3 in terms of the other variables. Use this expression to eliminate N_3 from the equation for dN_2 / dt. Solve this new equation for the ratio N_2 / N_1.

e. To reach a situation where there is gain, we must have a population inversion, i.e. $N_2 - \frac{g_2}{g_1} N_1 > 0$ or $\frac{N_2 g_1}{N_1 g_2} > 1$. In the limit where the pump rate is very large, show that it is possible to achieve gain provided that $A_{21}\tau_c < 1$.