Reading Today: 9.4, 9.5

Monday: Review
7.3, 8, 9
(mostly 9.5)

Dispersion

A material is dispersive if ϵ, μ, σ_c depend on ω (f, λ)

σ_c: conductivity

Why would it depend on ω?

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

In the book he treat bound electrons as electrons stuck to atoms by springs.

Diff eq. is

$$m\ddot{\vec{x}} + m\gamma\dot{\vec{x}} + m\omega_0^2 \vec{x} = \frac{q}{m}\vec{E}_0 e^{-i\omega t}$$

1-D $$m\ddot{x} + m\gamma\dot{x} + \underbrace{m\omega_0^2}_{k} x = \frac{q}{m} E_0 e^{-i\omega t}$$

ω_0: characteristic osc. freq. for that e^-.

Solution: $$\tilde{x}(t) = \frac{q/m}{\omega_0^2 - \omega^2 - i\gamma\omega} \tilde{E}_0 e^{-i\omega t}$$

$$\vec{P} = x(t)\, q$$

$$\tilde{\chi} = \frac{\vec{P}}{\epsilon_0 \vec{E}} = \frac{q/m}{\epsilon_0(\omega_0^2 - \omega^2 - i\gamma\omega)} \; ; \; \vec{P} = \epsilon_0 \tilde{\chi} \vec{E}$$

$$\tilde{\epsilon} = \epsilon_0(1 + \tilde{\chi})$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 \vec{E} + \epsilon_0 \tilde{\chi} \vec{E} = \underbrace{\epsilon_0(1+\tilde{\chi})}_{\tilde{\epsilon}} \vec{E}$$

Birefringence:

In ordered materials (like crystals), electrons can move more freely in one direction than another

the consequence is:

$$\vec{E} = E_x\hat{x} + E_y\hat{y}$$

$$\vec{D} = \epsilon_0(E_x\hat{x} + E_y\hat{y}) + P_x\hat{x} + P_y\hat{y}$$

$$\tilde{\chi}_x = \frac{\tilde{P}_x}{\epsilon_0 \vec{E}_x} = \frac{q/m}{\epsilon_0(\omega_0^2 - \omega^2 - i\gamma\omega)}$$

So you can have diff. χ in diff directions

Mathematically, I want to keep the equation

$$\vec{D} = \tilde{\epsilon}\,\vec{E} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix} \vec{E}$$

$$D_x = \epsilon_{xx}E_x + \epsilon_{xy}E_y + \epsilon_{xz}E_z$$

Birefringence $\vec{D} = \begin{pmatrix} \epsilon_{xx} & & \emptyset \\ & \epsilon_{yy} & \\ \emptyset & & \epsilon_{zz} \end{pmatrix} \vec{E}$

Let's do the same thing, but for free electrons.

EM wave (plane) $B = \frac{E}{c}$

Let's ignore you.

$F_q = q\vec{E} + q\vec{v}\times\vec{B}$

$|\uparrow|$ $|\uparrow|$

qE, $\leq qvB = qE\frac{v}{c} \Rightarrow qE\frac{v}{c} < qE$

HO: $m\ddot{x} = qE_x - \gamma\dot{x}$ $(-m\omega_0^2 x)$

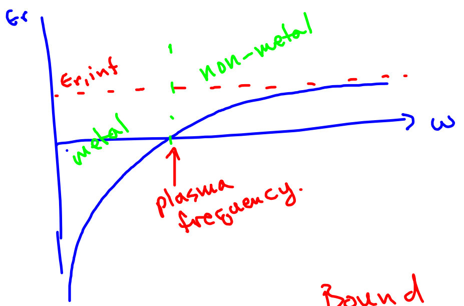
$\bullet e^-$

$$x(t) = \frac{q/m}{-\omega^2 - i\gamma\omega} e^{-i\omega t}$$

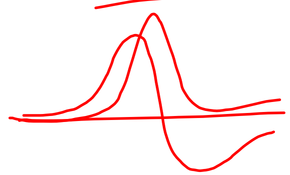
$$P_x(t) = n_e q x(t) = \frac{n_e q^2/m}{-\omega^2 - i\gamma\omega} e^{-i\omega t}$$

$$\tilde{\chi} = \frac{n_e q^2/m}{-\omega^2 - i\gamma\omega}$$

$$\tilde{\epsilon}_r = \tilde{\epsilon}_{r,inf} - \frac{n_e q^2/m}{\omega^2 + i\gamma\omega}$$



Bound



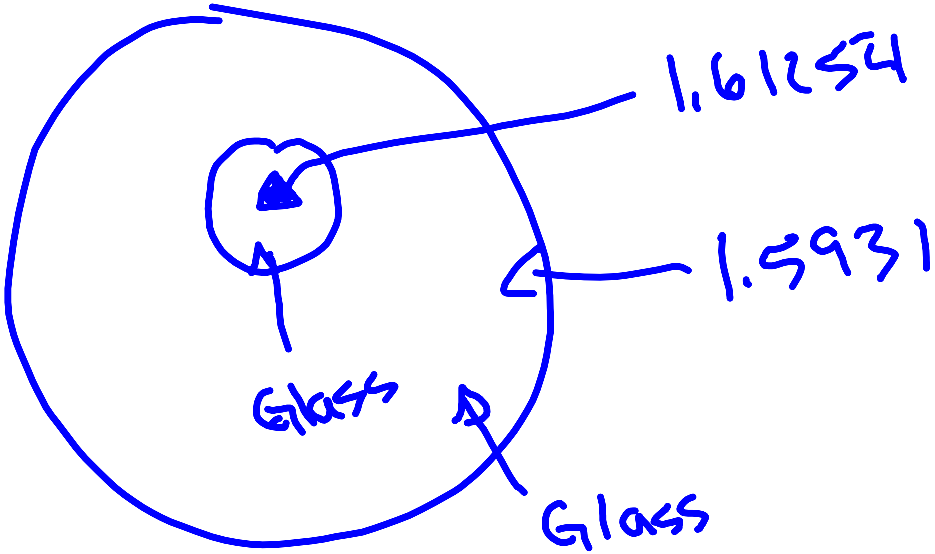
Cauchy Formula

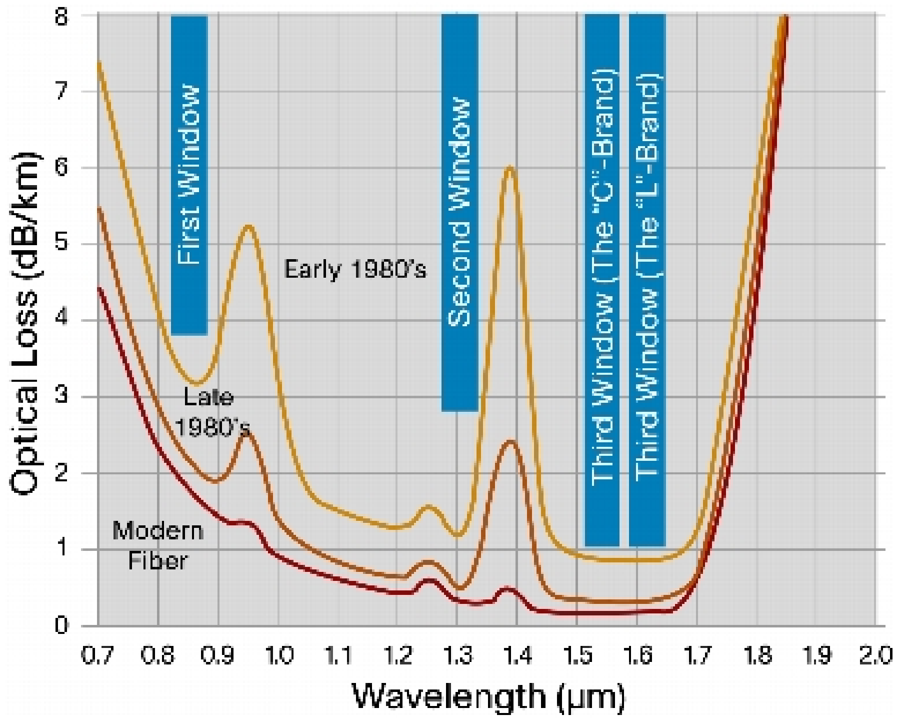
$$n = 1 + A\left(1 + \frac{B}{\lambda^2}\right)$$

Sellmeier Eqn

$$\epsilon(\lambda) = 1 + \frac{B_1\lambda_0^2}{\lambda_0^2 - C_1} + \frac{B_2\lambda_0^2}{\lambda_0^2 - C_2} + \frac{B_3\lambda_0^2}{\lambda_0^2 - C_3}$$

Optical Fibers





$$dB = 10 \log \left(\frac{P_{out}}{P_{in}} \right)$$

$$dB = 10 \log (.1)$$

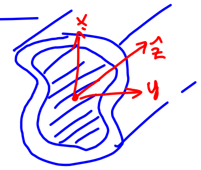
$$= -10 dB$$

Done

Guided waves

If walls are very good conductors.

$\vec{E}_{\parallel} = \emptyset \qquad B_{\perp} = \emptyset$



Maxwell's eqns:

$\vec{\nabla}\cdot\vec{E} = \emptyset \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\vec{\nabla}\cdot\vec{B} = \emptyset \qquad \vec{\nabla}\times\vec{B} = \mu\epsilon \frac{\partial \vec{E}}{\partial t}$

If propagation is in z-direction, then

$\vec{E} = \vec{E}_0 e^{i(k_z z - \omega t)} \qquad \vec{B} = \vec{B}_0 e^{i(k_z z - \omega t)}$

depend only on x,y

1 $\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = +i\omega B_z$

2 $\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = +i\omega B_x$

3 $\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = +i\omega B_y$

4 $\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = -\frac{i\omega}{v^2} E_z$

5 $\frac{\partial B_z}{\partial y} - \underbrace{\frac{\partial B_y}{\partial z}}_{ik_z B_y} = -\frac{i\omega}{v^2} E_x$

6 $\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} = -\frac{i\omega}{v^2} E_y$

$\frac{\partial}{\partial z} \Rightarrow ik_z$

$3,5 \Rightarrow \; ik_z E_x - \frac{\partial E_z}{\partial x} = \frac{i\omega}{ik_z}\left[\frac{\partial B_z}{\partial y} + \frac{i\omega}{v^2} E_x\right]$

$$E_x = \frac{i}{\omega^2/v^2 - k_z^2}\left(k_z \frac{\partial E_z}{\partial x} + \omega \frac{\partial B_z}{\partial y}\right)$$

Also:

$$E_y = \frac{i}{\omega^2/v^2 - k_z^2}\left(k_z \frac{\partial E_z}{\partial y} - \omega \frac{\partial B_z}{\partial x}\right)$$

$$B_x = \frac{i}{\omega^2/v^2 - k_z^2}\left(k_z \frac{\partial B_z}{\partial x} - \frac{\omega}{v^2} \frac{\partial E_z}{\partial y}\right)$$

$$B_y = \frac{i}{\omega^2/v^2 - k_z^2}\left(k_z \frac{\partial B_z}{\partial y} + \frac{\omega}{v^2} \frac{\partial E_z}{\partial x}\right)$$

$$E_x = \frac{i}{\omega^2/v^2 - k_z^2}\left(k_z \frac{\partial E_z}{\partial x} + \omega \frac{\partial B_z}{\partial y}\right)$$

Also:

$$E_y = \frac{i}{\omega^2/v^2 - k_z^2}\left(k_z \frac{\partial E_z}{\partial y} - \omega \frac{\partial B_z}{\partial x}\right)$$

$$B_x = \frac{i}{\omega^2/v^2 - k_z^2}\left(k_z \frac{\partial B_z}{\partial x} - \frac{\omega}{v^2}\frac{\partial E_z}{\partial y}\right)$$

$$B_y = \frac{i}{\omega^2/v^2 - k_z^2}\left(k_z \frac{\partial B_z}{\partial y} + \frac{\omega}{v^2}\frac{\partial E_z}{\partial x}\right)$$

Define $\vec{E}_T, \vec{B}_T = E_x\hat{x} + E_y\hat{y},\ B_x\hat{x} + B_y\hat{y}$

Define an operator $\vec{\nabla}_T \equiv$ such that $\vec{\nabla} = \vec{\nabla}_T + \vec{\nabla}_z$

$$\vec{E}_T = \frac{i}{\frac{\omega^2}{v^2} - k_z^2}\left[k_z\left[\frac{\partial E_z}{\partial x}\hat{x} + \frac{\partial E_z}{\partial y}\hat{y}\right] + \omega\left[\frac{\partial B_z}{\partial y}\hat{x} - \frac{\partial B_z}{\partial x}\hat{y}\right]\right]$$

$$\vec{\nabla}_T \cdot \vec{\nabla}_T = \nabla_T^2 = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y}\right)^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$\vec{\nabla}_T \cdot (\vec{\nabla}_T \times \vec{v}) = \phi$$

You can show that the above eqn for $\vec{E}_T$ can be written as

$$\vec{E}_T = \frac{i}{\frac{\omega^2}{v^2} - k_z^2}\left[k_z \vec{\nabla}_T E_z + \omega \vec{\nabla}_T \times \vec{B}_z\right]$$

$$\vec{B}_T = \frac{i}{\frac{\omega^2}{v^2} - k_z^2}\left[k \vec{\nabla}_T B_z - \frac{\omega}{v^2}\vec{\nabla}_T \times \vec{E}_z\right]$$

We're shooting for is:

$$\nabla_T^2 E_z + \left(\frac{\omega^2}{v^2} - k_z^2\right)E_z = \phi$$

$$\nabla_T^2 B_z + \left(\frac{\omega^2}{v^2} - k_z^2\right)B_z = \phi$$

Attachments

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