Problem 7.26

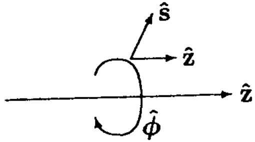
(a) $W = \frac{1}{2}LI^2$. $L = \mu_0 n^2 \pi R^2 l$ (Prob. 7.22) $\boxed{W = \frac{1}{2}\mu_0 n^2 \pi R^2 l I^2}$.

(b) $W = \frac{1}{2}\oint (\mathbf{A}\cdot\mathbf{I})\,dl$. $\mathbf{A} = (\mu_0 n I/2)R\,\hat{\boldsymbol{\phi}}$, at the surface (Eq. 5.70 or 5.71). So $W_1 = \frac{1}{2}\frac{\mu_0 n I}{2}RI\cdot 2\pi R$, for one turn. There are nl such turns in length l, so $W = \frac{1}{2}\mu_0 n^2 \pi R^2 l I^2$. ✓

(c) $W = \frac{1}{2\mu_0}\int B^2\,d\tau$. $B = \mu_0 n I$, inside, and zero outside; $\int d\tau = \pi R^2 l$, so $W = \frac{1}{2\mu_0}\mu_0^2 n^2 I^2 \pi R^2 l = \frac{1}{2}\mu_0 n^2 \pi R^2 l I^2$. ✓

(d) $W = \frac{1}{2\mu_0}\left[\int B^2\,d\tau - \oint (\mathbf{A}\times\mathbf{B})\cdot d\mathbf{a}\right]$. This time $\int B^2\,d\tau = \mu_0^2 n^2 I^2 \pi (R^2 - a^2) l$. Meanwhile, $\mathbf{A}\times\mathbf{B} = 0$ outside (at $s = b$). Inside, $\mathbf{A} = \frac{\mu_0 n I}{2}a\,\hat{\boldsymbol{\phi}}$ (at $s = a$), while $\mathbf{B} = \mu_0 n I\,\hat{\mathbf{z}}$.

$\mathbf{A}\times\mathbf{B} = \frac{1}{2}\mu_0^2 n^2 I^2 a \underbrace{(\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}})}_{\hat{\mathbf{s}}}$



points *inward* ("out" of the volume)

$\oint (\mathbf{A}\times\mathbf{B})\cdot d\mathbf{a} = \int \left(\frac{1}{2}\mu_0^2 n^2 I^2 a\hat{\mathbf{s}}\right)\cdot\left[a\,d\phi\,dz(-\hat{\mathbf{s}})\right] = -\frac{1}{2}\mu_0^2 n^2 I^2 a^2 2\pi l$.

$W = \frac{1}{2\mu_0}\left[\mu_0^2 n^2 I^2 \pi (R^2 - a^2) l + \mu_0^2 n^2 I^2 \pi a^2 l\right] = \frac{1}{2}\mu_0 n^2 I^2 R^2 \pi l$. ✓

Problem 7.33

(a) $\mathbf{J}_d = \epsilon_0 \frac{\mu_0 I_0 \omega^2}{2\pi}\cos(\omega t)\ln(a/s)\,\hat{\mathbf{z}}$. But $I_0\cos(\omega t) = I$. So $\boxed{\mathbf{J}_d = \frac{\mu_0\epsilon_0}{2\pi}\omega^2 I \ln(a/s)\,\hat{\mathbf{z}}.}$

(b) $I_d = \int \mathbf{J}_d\cdot d\mathbf{a} = \frac{\mu_0\epsilon_0\omega^2 I}{2\pi}\int_0^a \ln(a/s)(2\pi s\,ds) = \mu_0\epsilon_0\omega^2 I\int_0^a (s\ln a - s\ln s)ds$

$= \mu_0\epsilon_0\omega^2 I\left[(\ln a)\frac{s^2}{2} - \frac{s^2}{2}\ln s + \frac{s^2}{4}\right]\Big|_0^a = \mu_0\epsilon_0\omega^2 I\left[\frac{a^2}{2}\ln a - \frac{a^2}{2}\ln a + \frac{a^2}{4}\right] = \boxed{\frac{\mu_0\epsilon_0\omega^2 I a^2}{4}}.$

(c) $\boxed{\frac{I_d}{I} = \frac{\mu_0\epsilon_0\omega^2 a^2}{4}.}$ Since $\mu_0\epsilon_0 = 1/c^2$, $I_d/I = (\omega a/2c)^2$. If $a = 10^{-3}$ m, and $\frac{I_d}{I} = \frac{1}{100}$, so that $\frac{\omega a}{2c} = \frac{1}{10}$, $\omega = \frac{2c}{10a} = \frac{3\times 10^8\text{ m/s}}{5\times 10^{-3}\text{ m}}$, or $\omega = 0.6\times 10^{11}$/s $= \boxed{6\times 10^{10}\text{/s};}$ $\nu = \frac{\omega}{2\pi} \approx 10^{10}$ Hz, or 10^4 megahertz. (This is the *microwave* region, *way* above radio frequencies.)