1. Given

(1) $my'' + ky = 0$, $y(0) = -1$, $y'(0) = +1$

a) Let $\frac{dy}{dt} = y' = V$ then (1) becomes,

$$\frac{dV}{dt} = V' = \frac{-ky}{m} \Rightarrow \begin{aligned} \frac{dy}{dt} &= V \\ \frac{dV}{dt} &= -\frac{k}{m}y \end{aligned} \Leftrightarrow \frac{d}{dt}\begin{bmatrix} y \\ V \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix}}^{A} \begin{bmatrix} y \\ V \end{bmatrix}$$

where the initial cond. gives

$$\vec{Y}(0) = \begin{bmatrix} y(0) \\ V(0) \end{bmatrix} = \begin{bmatrix} -1 \\ +1 \end{bmatrix}$$

b)

$$\det(A - \lambda I) = \lambda^2 + \frac{k}{m} = 0 \Rightarrow \lambda = \pm\sqrt{\frac{-k}{m}} =$$

$$= \pm\sqrt{\left|\frac{k}{m}\right|}\, i = \pm \omega_0 i$$

Case $\lambda_1 = \omega_0 i$

$$(A - \lambda_1 I)\vec{V} = \begin{bmatrix} -\omega_0 i & 1 \\ -\frac{k}{m} & -\omega_0 i \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \omega_0 i v_1 = v_2 \Rightarrow \vec{V}^{(1)} = \begin{bmatrix} 1 \\ \omega_0 i \end{bmatrix}$$

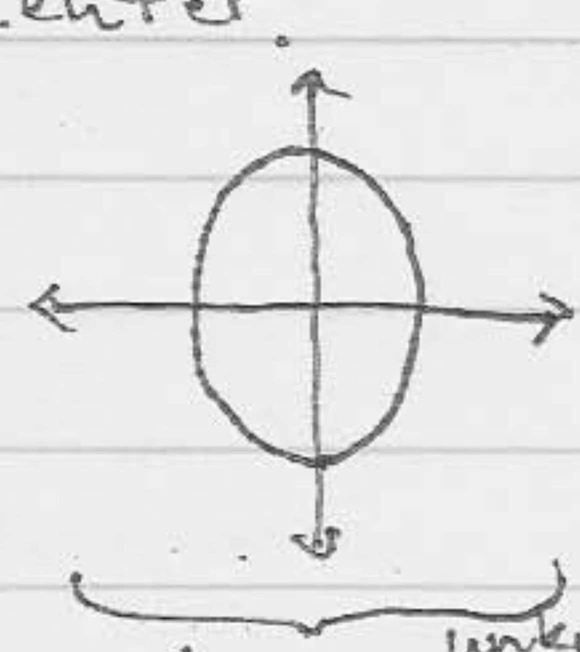
$$\Rightarrow \vec{Y}(t) = k_1 \begin{bmatrix} 1 \\ \omega_0 i \end{bmatrix} e^{\omega_0 i t} + k_2 \begin{bmatrix} 1 \\ -\omega_0 i \end{bmatrix} e^{-\omega_0 i t}$$

or

$$\vec{Y}_{Real}(t) = k_1 \begin{bmatrix} \cos(\omega_0 t) \\ -\omega_0 \sin(\omega_0 t) \end{bmatrix} + k_2 \begin{bmatrix} \sin(\omega_0 t) \\ \cos(\omega_0 t) \end{bmatrix}$$

$$\vec{Y}(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \Rightarrow k_1 = -1 \quad k_2 = 1$$

(c) Equilibrium is a Center.



$\underbrace{\qquad\qquad}_{\text{Ellipse direction}}$ unknown