Phys 361 Homework 1

1a) In Phys 200 we used Gauss's Law to determine the field made by an infinite, uniform sheet of charge. With Gauss's Law it was trivial. I told you that it was quite doable with brute force integration as well, and that someday you'd probably be forced to do it. Today's the day. Pick an arbitrary point some distance z above an infinite sheet in the xy plane with charge density Φ. Calculate the electric field at that location using brute force integration. You may use tables/Mathematica for integrals that are beyond simple u-subs, and you may appeal to symmetry, but don't just take the whole integral, jam it into a machine and say "see, it works!"

1b) Now forget all that and give me a compelling *geometric* argument as to why the field should be independent of z. There should be lots of words and probably a picture here. It's quite possible to frame this so that even a non-physics major in Phys 200 can understand it, meaning that it need not be a formal proof.

1c) In chapter 3, example 4, Pollack and Stump prove that the field inside a spherical shell of uniform charge is zero, on or off the center of the sphere. They do it by brute force. You can justify why this should be so geometrically in much the same way as you handled 1b. Do it. Remember: Words and pictures.

1d) Next, let's switch up the dimensionality. Consider a uniform ring of charge (that is, a 1-D circle of charge of some radius sitting in a 2-D plane). It should be fairly obvious from the symmetry of the situation that the field at the dead center of the ring is zero. But is that true at points inside the circle but off-center? Based on your experiences in 1b and 1c you should be able to consider the geometry of the situation and decide yes or no, and justify your answer. If you're stumped, find the answer via integration and then retroactively justify why it must be so.

1e) In Phys 200, we say that a non-infinite sheet of charge can be considered infinite as long as it is "large enough." Consider a finite square of charge of side lengths L and some point P a distance z above the center of the square. Let E_{finite} represent the electric field at that point. Let $E_{infinite}$ represent the field that an infinite sheet would produce at the same point. Use whatever software package you prefer to find the ratio z/L at which $E_{finite}/E_{infinite}$ is 0.9. Comment on what you've learned about "large enough" in this context.

2) Use Gauss's Law to figure out the electric field inside and outside of an infinite cylinder of radius *R* and filled with a uniform volume charge density ρ_0. Start from the integral form of Gauss's Law and *thoroughly* justify *every* step. Did a vector become a scalar, or a dot product go away? There better be a good reason. Did you decide that the field is strictly radial? That's not free, you know. And if anyone simply waves their hands and says "by symmetry, everything magically works," well, let's just say *don't do that*.

There should be substantial writing on this problem. There are a number of subtleties involved that require genuine thought. I have only a passing interest in the answer itself, since we did this problem many times in Phys 200. Note that my solution in the answer key spans more than two pages. Yours need not be quite so wordy, but you need to attend to all the different steps involved in doing this problem right.

3) Show that the integral and differential forms of Gauss's Law are equivalent. Go both ways (start from the differential form and derive the integral form, then start from the integral form and go back). This one should be relatively short.

4) (based on Problem 3.23 in Pollack & Stump)

a) Consider a spherically-symmetric, nonuniform charge distribution with volume charge density $\rho(r) = Cr$ for $r < a$, and no other charge anywhere. Find the field and potential both inside and outside the sphere. If you use Gauss's Law to find an electric field, you may dispense with most of the detailed explanation that I required in the last assignment.

b) Explicitly check whether the field and potential are continuous at $r = a$.

c) Now suppose that there's *also* a uniform surface charge σ_0 at $r = a$. Find the new field and potential inside and outside the sphere. Check whether they're continuous at $r = a$, and comment on whether that makes sense. Either of "yes, it makes sense" or "no, it doesn't" is perfectly acceptable as long as you say something thoughtful to go along with it.

5) Take a *hollow* spherical shell with radius *R* and total charge *Q* distributed uniformly across the surface. Calculate the self-energy *U* of this object. Do it two different ways. First, explicitly consider the electrostatic interactions between the various bits of charge in the object (so starting with eqn. 3.80 or 3.81 in Pollack and Stump, or 2.43 in Griffiths). Second, try it strictly in terms of electric fields (so start from 3.83 in Pollack and Stump, or 2.45 in Griffiths). Note that these two perspectives are very distinct physically. Take the limit as *R* approaches zero and see what happens. Is this bad? Why?

6a) Coulomb's law goes like $1/r^2$. But why should it do that instead of, say, $1/r^3$? Come up with a geometrical argument for why Coulomb's law should have the dependence that it does. You can't prove it from first principles, so if you have to assume something (something that makes intuitive sense to you), that's okay. I like to think in terms of flux and the dimensionality of our universe, but there may be other approaches.

b) Let's now suppose that we're in Flatland. If you've never heard of Flatland, it's a two-dimensional world inhabited by sentient polygons. With your argument from (1a) in mind, what kind of Coulomb's law dependence would you expect Flatlanders to observe? Note that this would probably also apply to gravity, since it's structurally almost identical.

c) Now let's suppose that Flatlander physicists get around to measuring the law of gravity and find that it actually goes like $1/r^2$, instead of what they expected. What might this mean about their universe? If you're stumped, you could always read Flatland or a summary thereof for a hint.

d) Finally, with all the previous in mind, consider those zany string theorists that keep claiming that our three-dimensional world might actually have as many as 26 dimensions. Read this:

http://www.npl.washington.edu/eotwash/sr

and comment on its pertinence to what we've done in this problem so far.