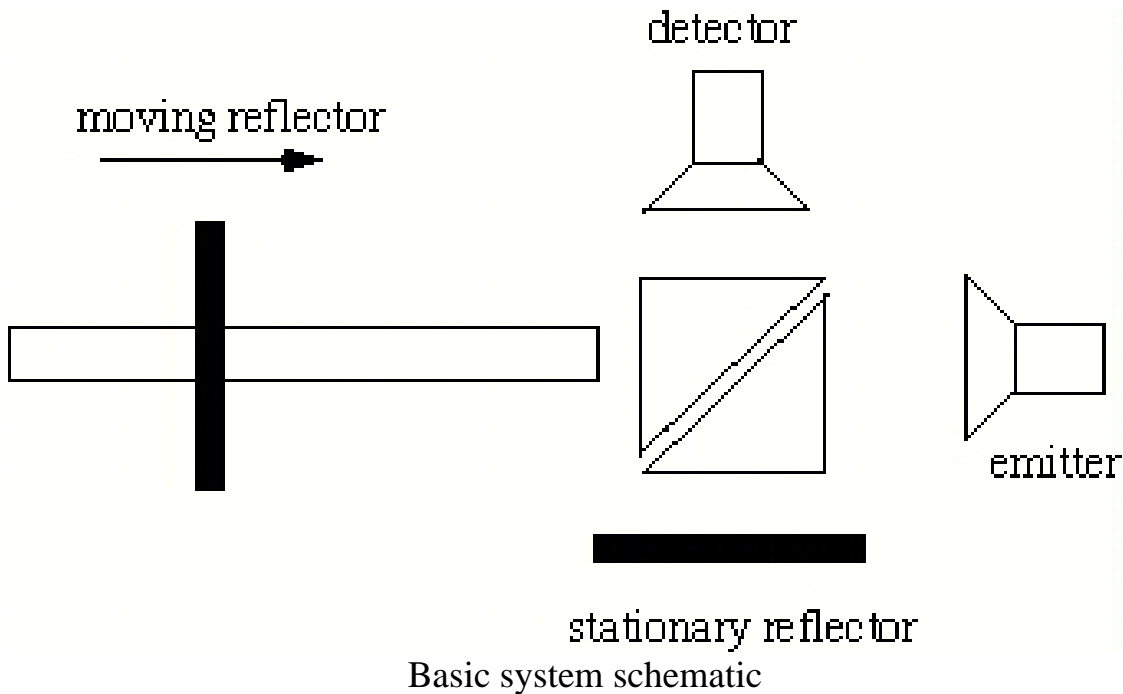


Microwave Doppler Shift

In this lab, we want to build a classical Michelson interferometer, with a twist: one of the arms is mobile; the reflector is mounted on a car that moves down an air track. For our source, we will use microwaves, so we can expect a fringe spacing on the order of millimeters (what fringe spacing would we expect for optical frequencies?). The basic system configuration is shown in the figure.



With this apparatus we can capture a fringe pattern in time. The detected field is the sum of the fields in the two arms. If we assume that the beamsplitter is 50/50, we can calculate the fringe pattern that we can expect to measure. The field entering each arm can be written as

$$0.5 E_o \cos \omega t \quad (1)$$

or, in complex-exponential notation,

$$0.5 E_o e^{i\omega t} . \quad (2)$$

If the path lengths $(l_2 - l_1)$ between the two arms give rise to a phase difference $\Delta = (2\pi/\lambda) \cdot 2(l_2 - l_1)$, we have at the detector

$$E_{\text{det}} = 0.5 E_o e^{i\omega t} + 0.5 E_o e^{i(\omega t + \Delta)} . \quad (3)$$

If we factor $0.5e^{i(\omega t + \Delta/2)}$ from both terms, we have

$$E_{\text{det}} = 0.5 e^{i(\omega t + \Delta/2)} (e^{-i\Delta/2} + e^{i\Delta/2}) = e^{i(\omega t + \Delta/2)} \cos(\Delta/2). \quad (4)$$

The detector follows only the average electric field; that is, it cannot detect the angular frequency ω of the microwave signal and instead averages the field over many cycles, so the measured field is

$$E_{\text{meas}} = \cos(\Delta/2). \quad (5)$$

Δ , however, is time dependent:

$$\Delta \rightarrow \Delta(t) \propto v, \quad (6)$$

where v is the speed of the cart, so you can see fringes on the oscilloscope. The measured (average) field is a maximum every time $\Delta = 2m\pi$, where m is an integer (positive or negative). How can you measure velocity by counting fringes?

Suggestions:

1. Capture and evaluate a series of interference patterns for a variety of speeds and distances. Use these measurements to calculate the velocity of the source.
2. Measure the cart speed using a stopwatch and ruler. Do the speeds measured using these two methods agree? What is the standard uncertainty of a measurement, presuming that you counted N fringes? Hint: The 100 % confidence interval of the fringe count is $N/2$; calculate the resulting uncertainty of the velocity.
3. Alternate way to look at the interference pattern. Assume that the frequency of the beam in the moving arm is Doppler-shifted. Show that you can derive the equation $m\lambda = 2d$, which describes the positions of the interference maxima.

Hint:

$$E_{\text{det}} = 0.5 E_o e^{i\omega t} + 0.5 E_o e^{i(\omega + \delta\omega)t}, \quad (7)$$

where $\delta\omega$ is the frequency shift due to the Doppler shift. Calculate $\delta\omega$ and show that you get the same result as you get by using Eq. (5).