FOWLES "MODERN OPTICS"

The angular size of the Airy disk is thus slightly larger than the corresponding value λ/b for the bright central band of the diffraction pattern of the rectangular aperture or slit. In Table 5.1 are listed the values of the first few maxima of the diffraction patterns of rectangular and circular apertures.

Optical Resolution The image of a distant point source formed at the focal plane of an optical-telescope lens or a camera lens is actually a Fraunhofer diffraction pattern for which the aperture is the lens opening. Thus the image of a composite source is a superposition of many Airy disks. The resolution of detail in the image therefore depends on the size of the individual Airy disks. If D is the diameter of the lens opening, then the angular radius of an Airy disk is approximately $1.22\ \lambda/D$. This is also the approximate minimum angular separation between two equal point sources such that they can be just barely resolved, because at this angular separation the central maximum of the image of one source falls on the first minimum of the other (Figure 5.14). This condition for optical resolution is known as

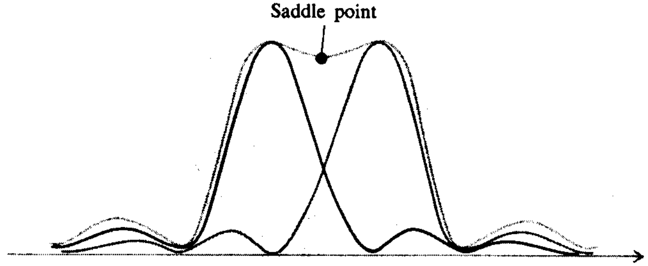

Figure 5.14. Rayleigh criterion.

the *Rayleigh criterion*. It is more convenient to use, in this case, than the Taylor criterion mentioned earlier.

In the case of the rectangular aperture, the minimum angular separation according to the Rayleigh criterion is just λ/b, where b is the width of the aperture. The intensity at the saddle point in this case is $8/\pi^2 = 0.81$ times the maximum intensity. The proof of this statement is left as an exercise.

The Double Slit Let us consider a diffracting aperture consisting of two parallel slits, each of width b and separated by a distance h (Figure 5.15). As with the single slit, we treat this case as a one-

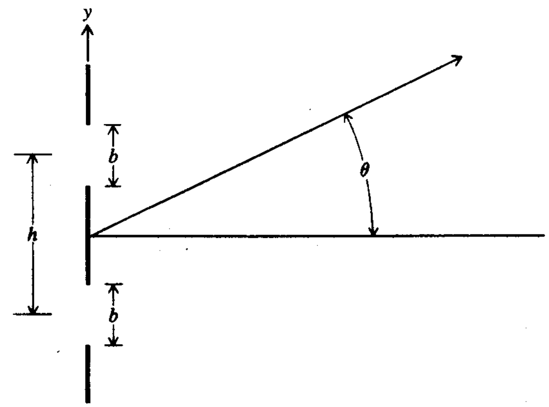

Figure 5.15. Double-slit aperture.

dimensional problem. The relevant diffraction integral is evaluated as follows:

$$\begin{aligned}\int_{\mathscr{A}} e^{iky\sin\theta}\,dy &= \int_0^b e^{iky\sin\theta}\,dy + \int_h^{h+b} e^{iky\sin\theta}\,dy\\ &= \frac{1}{ik\sin\theta}\left(e^{ikb\sin\theta} - 1 + e^{ik(h+b)\sin\theta} - e^{ikh\sin\theta}\right)\\ &= \left(\frac{e^{ikb\sin\theta}-1}{ik\sin\theta}\right)\left(1 + e^{ikh\sin\theta}\right)\\ &= 2b\,e^{i\beta}\,e^{i\gamma}\,\frac{\sin\beta}{\beta}\cos\gamma\end{aligned} \tag{5.26}$$

where $\beta = \frac{1}{2}kb\sin\theta$ and $\gamma = \frac{1}{2}kh\sin\theta$. The corresponding irradiance distribution function is

$$I = I_0\left(\frac{\sin\beta}{\beta}\right)^2\cos^2\gamma \tag{5.27}$$

The factor $(\sin\beta/\beta)^2$ is the previously found distribution function for a single slit. Here this factor constitutes an envelope for the interference fringes given by the term $\cos^2\gamma$. A plot is shown in Figure 5.16. Bright fringes occur for $\gamma = 0, \pm\pi, \pm2\pi$, and so forth. The angular separation between fringes is given by $\Delta\gamma = \pi$, or, approximately, in terms of the angle θ

$$\Delta\theta \approx \frac{2\pi}{kh} = \frac{\lambda}{h} \tag{5.28}$$

It is interesting to note that this is equivalent to the result of the analysis of Young's experiment [Equation (3.9)].