Geometric Optics and ABCD

Lenses and the imaging equation

Direct ray tracing

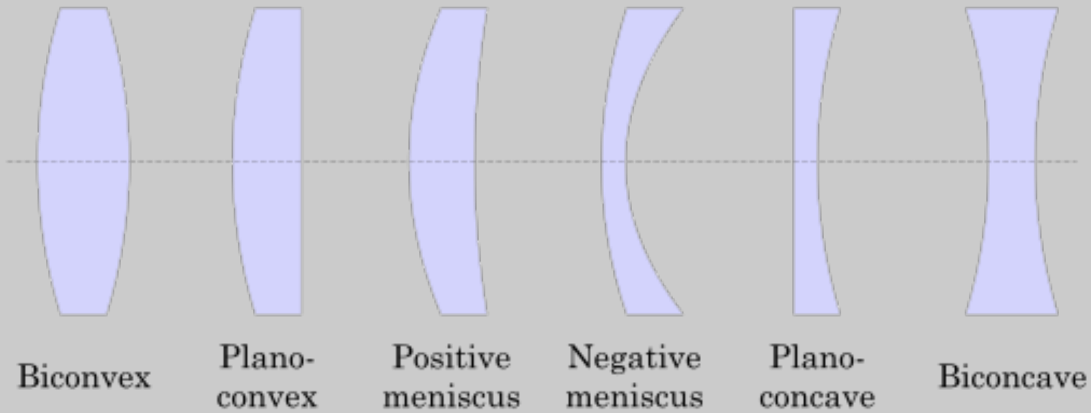
ABCD matrices

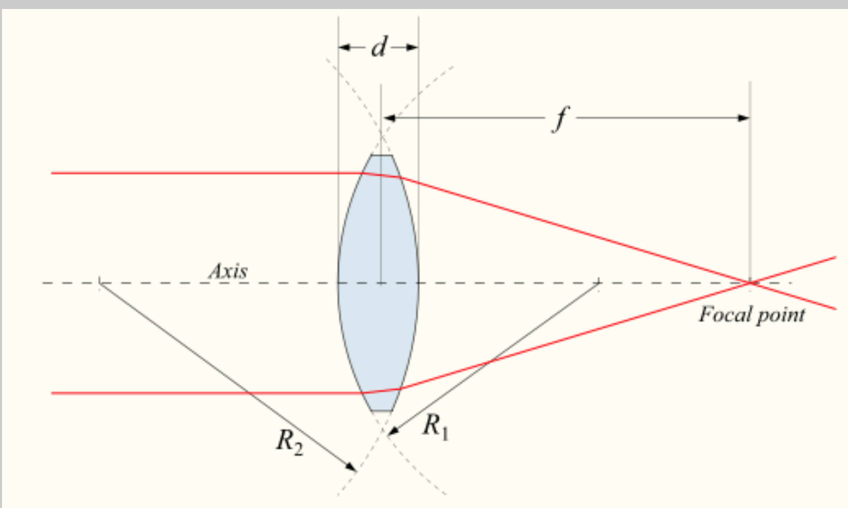
Geometric optics

- Approaches:
  - Paraxial calculations (assume small angle to optical axis)
    - Imaging equations: simple lens, Lensmakers' for thick lens
    - ABCD matrices for optical system
  - Non-paraxial, general case
    - Analytic calculation of aberrations: Zernike polynomials, Taylor-type expansions of wavefront error
    - Computer tracing (no approximations). Example: Zemax, Oslo
- Design procedure
  - Find existing design close to what could work for application
  - Paraxial trace with ray diagram
    - Calculate magnification, limiting apertures
  - Optimize with ABCD matrices or computer program
  - Analyze aberrations

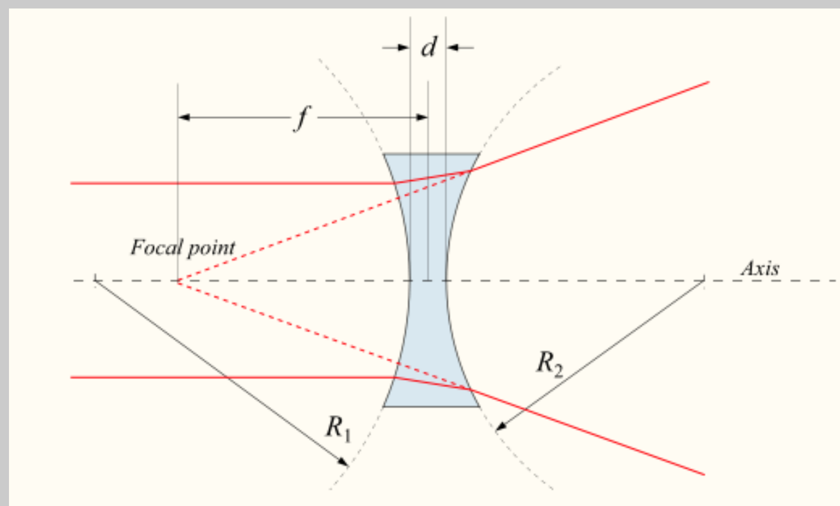
Geometric optics: lenses

For a nice summary, see Lens (optics) Wikipedia page
https://en.wikipedia.org/wiki/Lens_(optics)



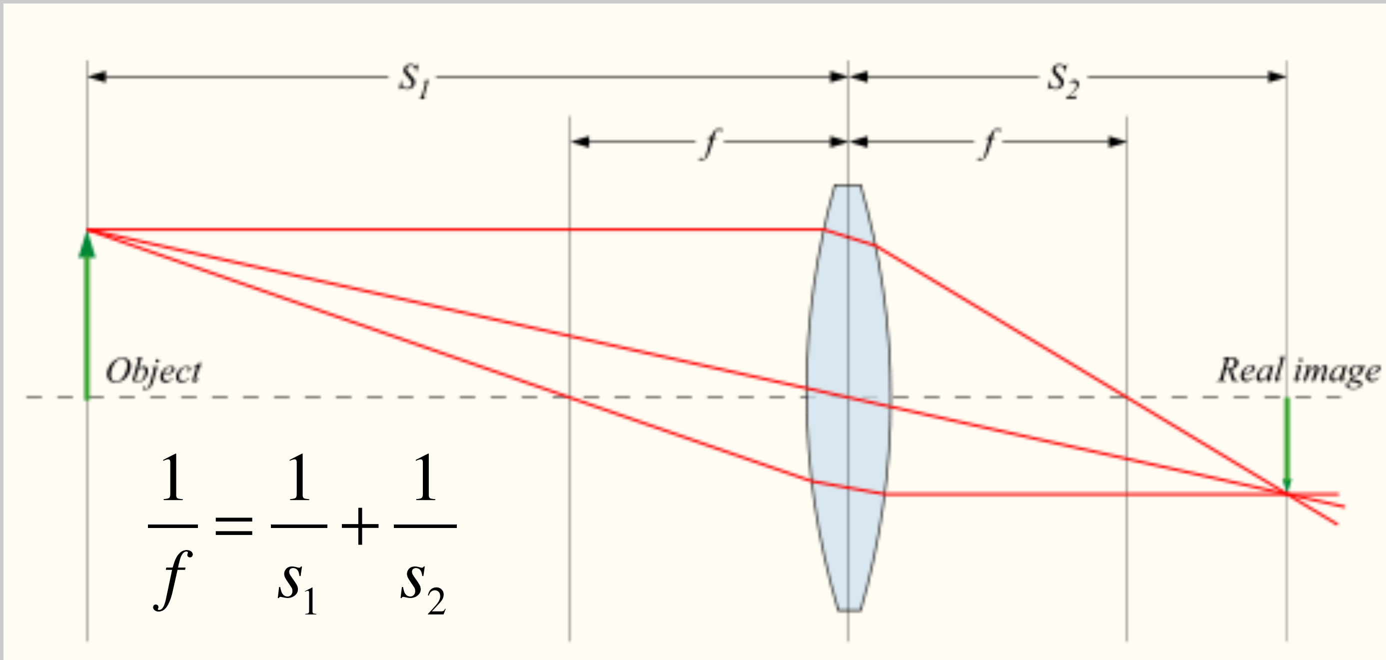


Positive (converging) lens



Negative (diverging) lens

Geometric optics: imaging equation



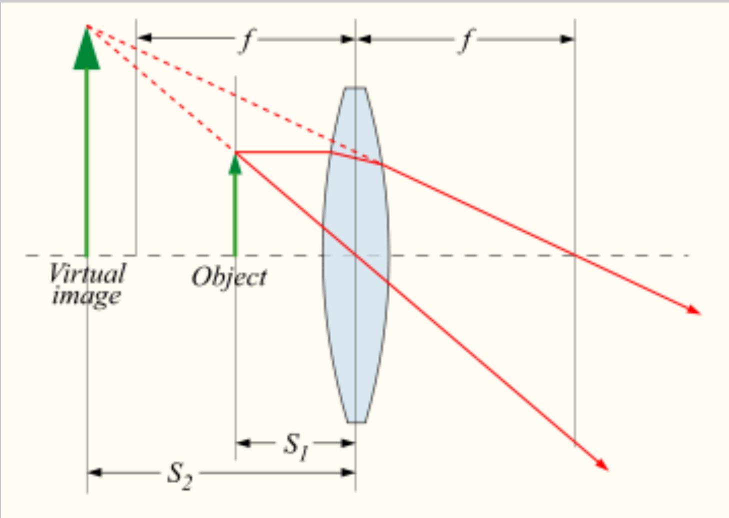
$$\frac{1}{f} = \frac{1}{s_1} + \frac{1}{s_2}$$

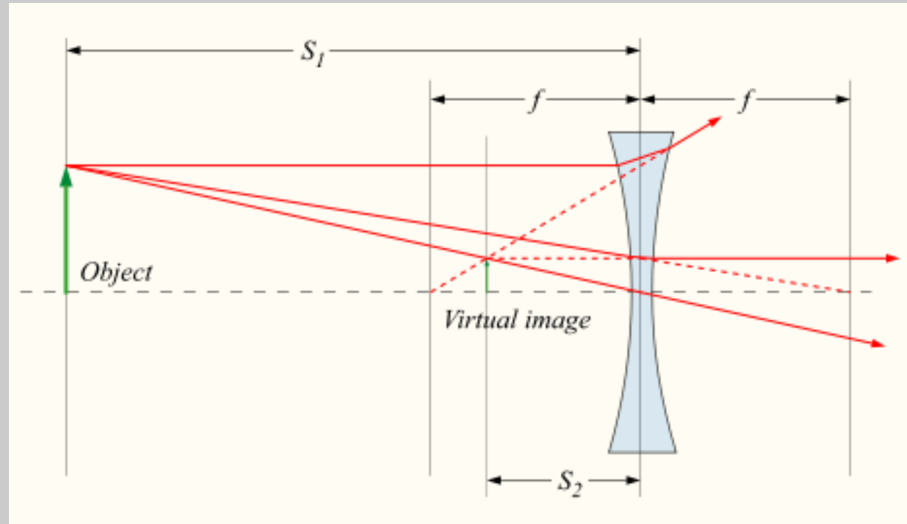
s_1, s_2 are positive as shown
s_1, s_2 = infinity: rays are collimated

$$M = -\frac{s_2}{s_1}$$

Magnification calculate with similar triangles
M < 0 inverted image
M = -1 when $s_1 = s_2 = 2f$, 1:1 imaging

Geometric optics: virtual images





s_1, s_2 < 0 if on the opposite side of lens

"virtual" image: position where image seems to come from

Concave lens: $f < 0$

Same convention for s_1, s_2

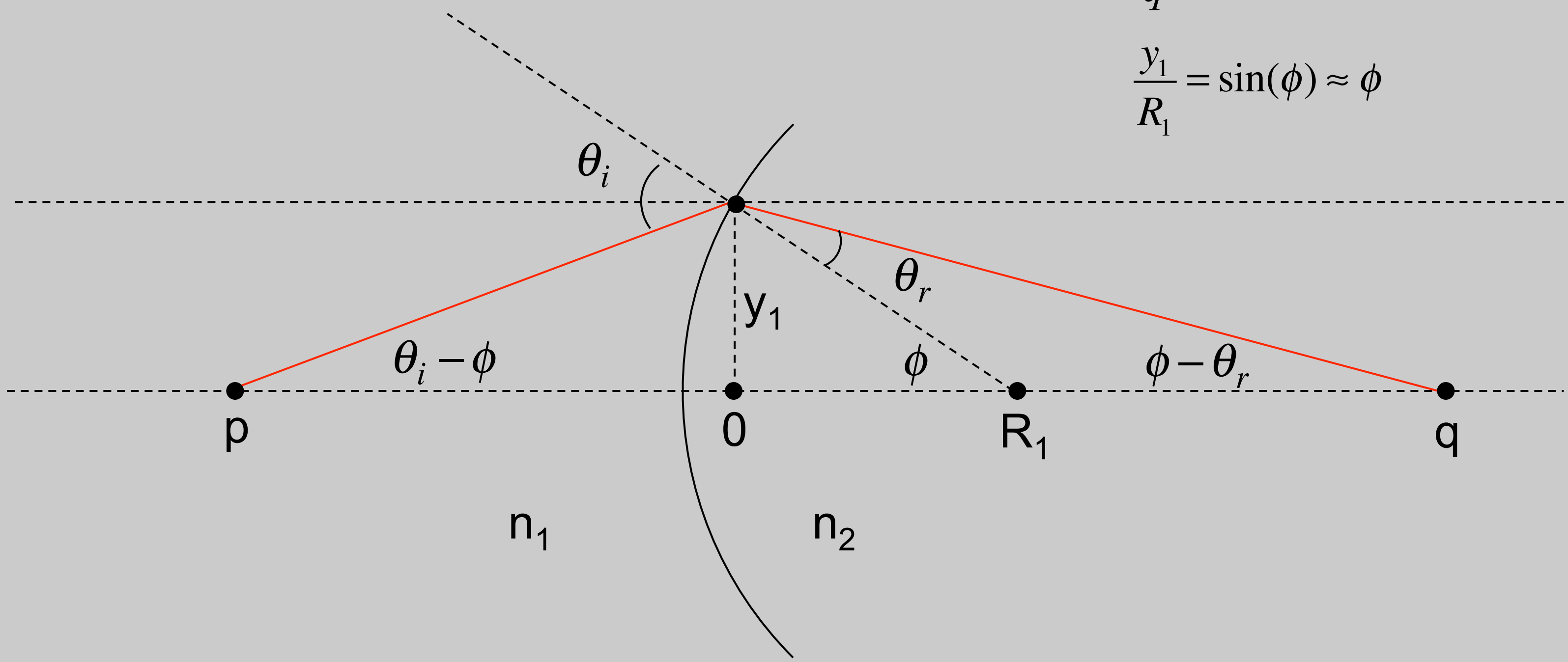
Raytracing: single curved interface

Snell: $n_1 \sin(\theta_i) = n_2 \sin(\theta_r)$

$$\frac{y_1}{p} = \tan(\theta_i - \phi) \approx \theta_i - \phi$$

$$\frac{y_1}{q} = \tan(\phi - \theta_r) \approx \phi - \theta_r$$

$$\frac{y_1}{R_1} = \sin(\phi) \approx \phi$$



$$\frac{n_2}{n_1} = \frac{\sin(\theta_i)}{\sin(\theta_r)} \approx \frac{\theta_i}{\theta_r} = \frac{\phi + \frac{y_1}{p}}{\phi - \frac{y_1}{q}} = \frac{\frac{y_1}{R_1} + \frac{y_1}{p}}{\frac{y_1}{R_1} - \frac{y_1}{q}} = \frac{\frac{1}{R_1} + \frac{1}{p}}{\frac{1}{R_1} - \frac{1}{q}}$$

In paraxial appx, y' s cancel

$$n_2\left(\frac{1}{R_1} - \frac{1}{q}\right) = n_1\left(\frac{1}{R_1} + \frac{1}{p}\right) \rightarrow \frac{1}{R_1}(n_2 - n_1) = \frac{n_2}{q} + \frac{n_1}{p}$$

Raytracing: two curved interfaces

- add second interface: R > 0 if center is to right
- assume $y_2=y_1$

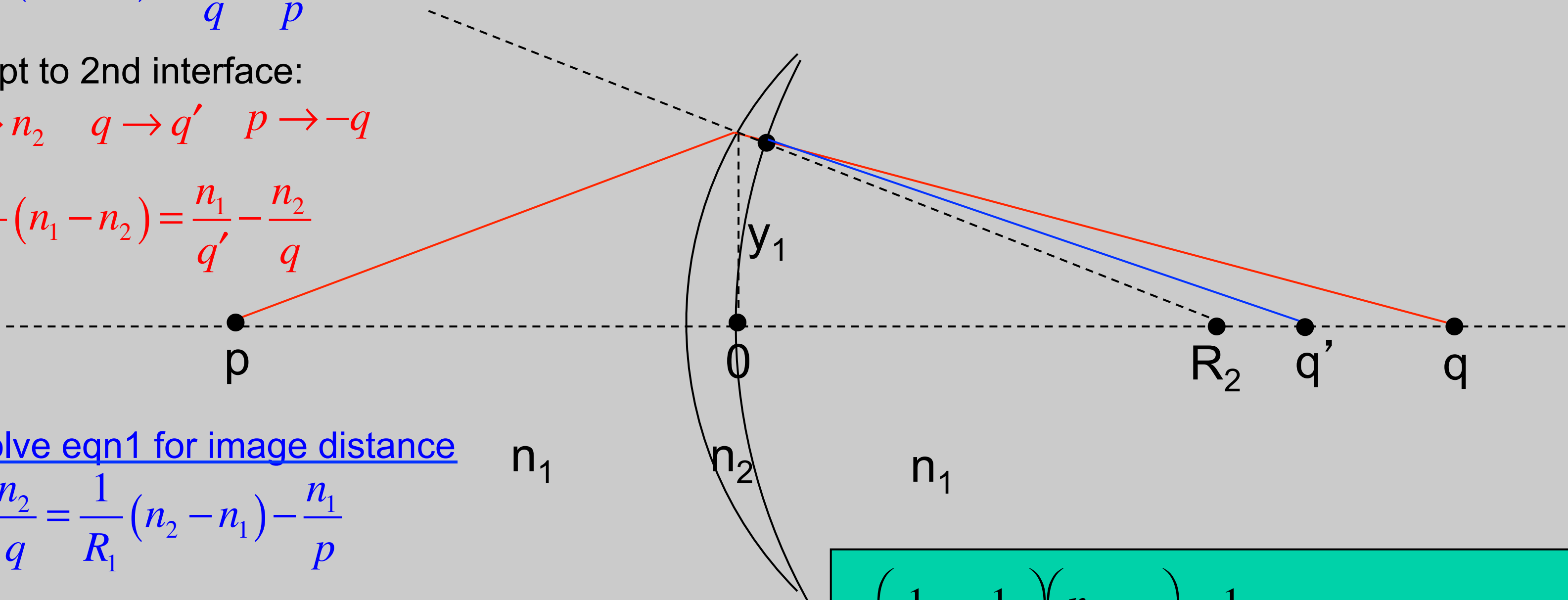
Eqn from 1st:

$$\frac{1}{R_1}(n_2-n_1)=\frac{n_2}{q}+\frac{n_1}{p}$$

Adapt to 2nd interface:

$$n_1 \leftrightarrow n_2 \quad q \rightarrow q' \quad p \rightarrow -q$$

$$\rightarrow \frac{1}{R_2}(n_1-n_2)=\frac{n_1}{q'}-\frac{n_2}{q}$$



Solve eqn1 for image distance

$$\rightarrow \frac{n_2}{q}=\frac{1}{R_1}(n_2-n_1)-\frac{n_1}{p}$$

$$\frac{1}{R_2}(n_1-n_2)=\frac{n_1}{q'}-\frac{1}{R_1}(n_2-n_1)+\frac{n_1}{p}$$

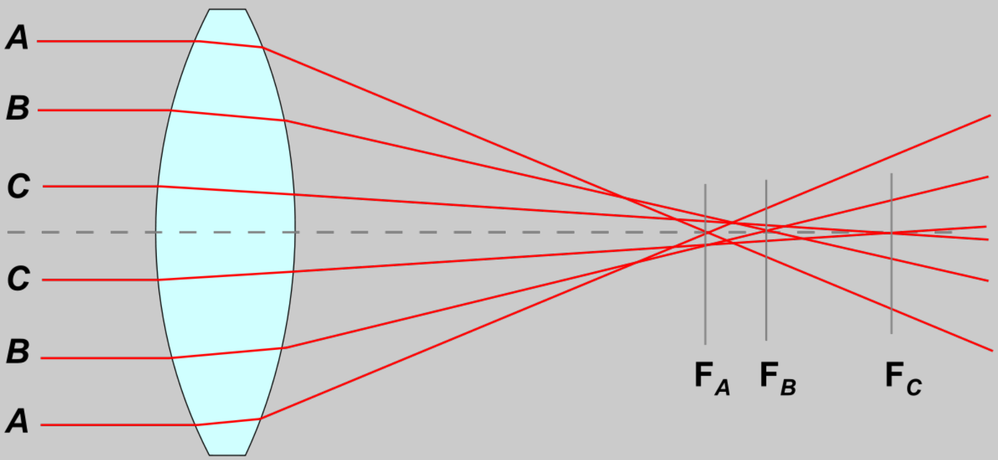
$$\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\left(\frac{n_2}{n_1}-1\right)=\frac{1}{q'}+\frac{1}{p}$$

$$\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\left(\frac{n_2}{n_1}-1\right)=\frac{1}{f}$$ Focal length (lensmaker's eqn)

$$\frac{1}{f}=\frac{1}{s_o}+\frac{1}{s_i}$$ Imaging equation

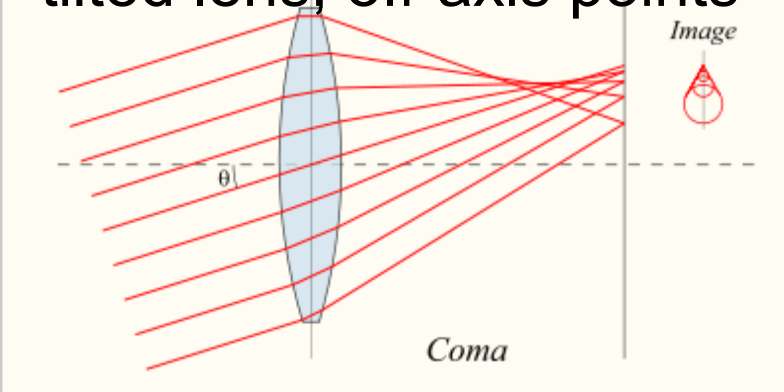
Aberrations

- For non-paraxial rays errors in how rays focus

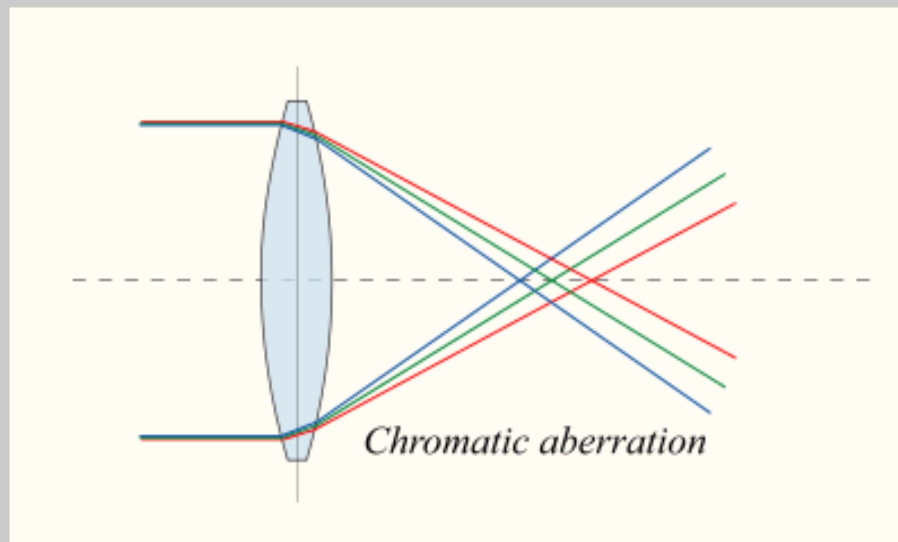


Spherical aberration

Coma:
tilted lens, off-axis points



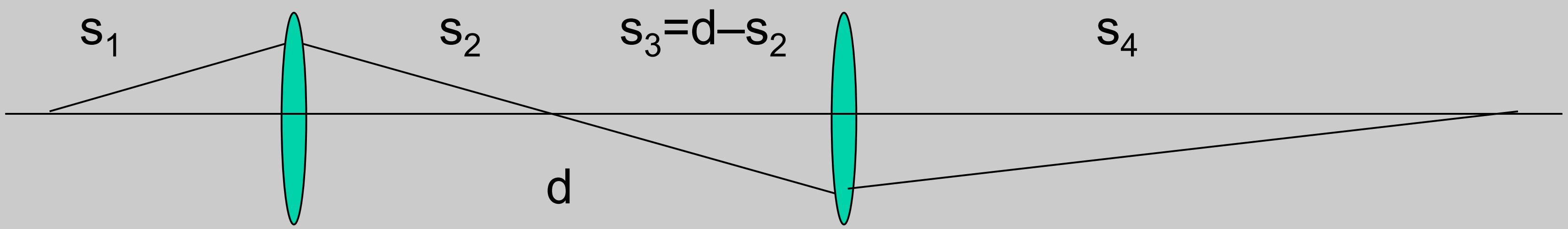
Coma



Chromatic aberration

Lens systems

- It is possible to cascade the lens imaging equation for multiple lenses:
  - Image from lens 1 is object for lens 2



- For more complicated systems, use a matrix method: ABCD matrices
  - Also good for resonators
  - Does not account for aberrations

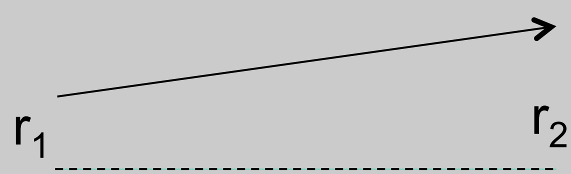
ABCD ray matrices

- Formalism to propagate rays through optical systems
  - Keep track of ray height r and ray angle $\theta = dr/dz = r'$
  - Treat this pair as a vector: $\begin{pmatrix} r \\ r' \end{pmatrix}$
  - Optical system will modify both the ray height and angle, e.g.

$$\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$$

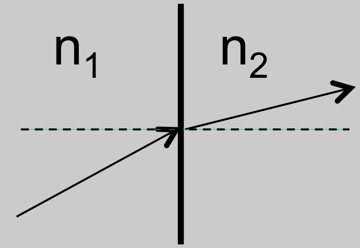
  - Successive ABCD matrices multiply from the left
- Translation

$$\begin{aligned} r_2 &= r_1 + L r_1' \\ r_2' &= r_1' \end{aligned} \quad \rightarrow \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$



Refraction in ABCD

- Translation: $\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$
- Flat interface



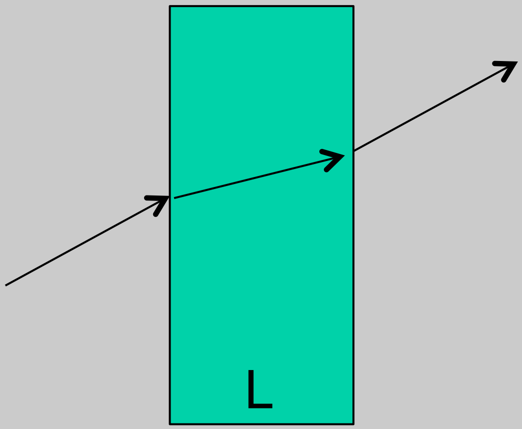
$$r_2 = r_1 \qquad n_1 \sin\theta_1 = n_2 \sin\theta_2 \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix}$$

$$n_1\theta_1 \approx n_2\theta_2$$

$$r_2' = \frac{n_1}{n_2} r_1'$$

Special case:
$n_1 = 1$, $n_2 = n$ $\rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1/n \end{pmatrix}$

- Window: calculate matrix



$$\rightarrow \begin{pmatrix} 1 & 0 \\ 0 & n \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1/n \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & n \end{pmatrix} \begin{pmatrix} 1 & L/n \\ 0 & 1/n \end{pmatrix} = \begin{pmatrix} 1 & L/n \\ 0 & 1 \end{pmatrix}$$

Effective thickness reduced by *n*

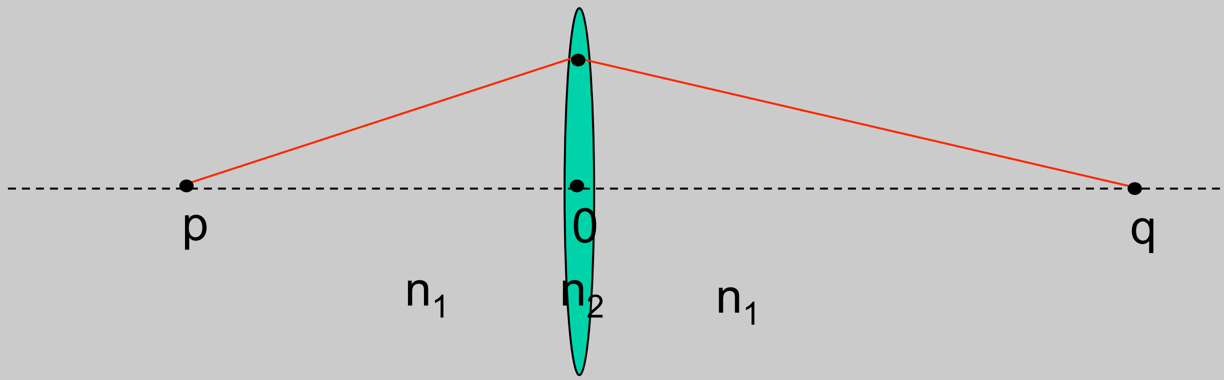
Curved surfaces in ABCD

- Thin lens: matrix computes transition from one side of lens to other

$$r_2 = r_1$$

$$r_1' = r_1 / p$$

$$r_2' = -r_1 / q$$



$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} \quad \rightarrow r_2' = -r_1\left(\frac{1}{f} - \frac{1}{p}\right) = -\frac{r_1}{f} + r_1' \quad \text{lens}\begin{pmatrix} A & B \\ C & D \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

- Spherical interface: radius *R*

$$\rightarrow \begin{pmatrix} 1 & 0 \\ \frac{n_1 - n_2}{n_2}\frac{1}{R} & \frac{n_1}{n_2} \end{pmatrix}$$