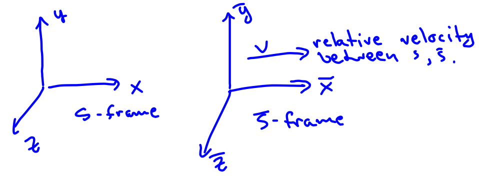
Today: G. 12.1, 12.2
Monday: G 12.3

Special Relativity.



General axioms of SR.
1) speed of light is constant for all observers.

$$\left.\begin{aligned} \bar{x} &= \gamma(x - vt) + x_0 \\ \bar{y} &= y + y_0 \\ \bar{z} &= z + z_0 \\ \bar{t} &= \gamma\left(t - \frac{v}{c^2}x\right) + t_0 \end{aligned}\right\} \quad \begin{aligned} x &= \gamma(\bar{x} + v\bar{t}) - x_0 \\ y &= \bar{y} - y_0 \\ z &= \bar{z} - z_0 \\ t &= \gamma\left(\bar{t} + \frac{v}{c^2}\bar{x}\right) - t_0 \end{aligned}$$

v is really v_x and is signed.

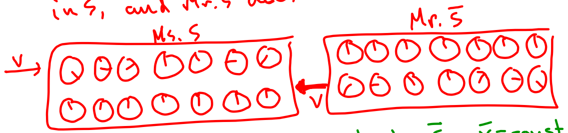
Griffiths says at $t=0$, $\bar{t}=0$, set $x=\bar{x}$, $y=\bar{y}$, $z=\bar{z}$

$$\Rightarrow \underline{x_0, y_0, z_0, t_0 = \emptyset}$$

Consequences:
Simultaneity: Let's say a guy in frame S say that something at $t=\emptyset$ happens exactly at the same time at $x=\pm a$.
What does Mr. $\bar{S}$ say?

Mr. $\bar{S}$ says event $(x=-a)$ happened at $\bar{t}=\frac{\gamma v}{c^2}a$
and event $(x=a)$ happened at $\bar{t}=-\frac{\gamma v}{c^2}a$.

Ms. S sets up a bunch of sinc'd clocks in S, and Mr. $\bar{S}$ does the same in $\bar{S}$.



Looking at a single clock in $\bar{S}$, $\bar{x}=$const
$\Delta t = \gamma \Delta\bar{t} \Rightarrow \Delta\bar{t} = \frac{1}{\gamma}\Delta t$ Time dialation.

Take $\Delta\bar{x}$ but with $\Delta t = 0$
$\Delta\bar{x} = \gamma\Delta x$

4-vectors

$x^\mu = (ct, x, y, z)$ ← contravariant
x^0, x^1, x^2, x^3

$x_\mu = (-ct, x, y, z)$ ← covariant.
x_0, x_1, x_2, x_3

$\beta = v/c$

$\bar{x}^\mu = \Lambda^\mu_\nu x^\nu$ $\quad \Lambda^\mu_\nu = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$\bar{x}^\mu = \sum_{\nu=0}^{3} \Lambda^\mu_\nu x^\nu$

Show gives the eqns. from the last page

$\bar{x}^0 = \gamma x^0 + (-\gamma\beta)x^1 + \cancel{0} x^2 + \cancel{0} x^3$
$\bar{x}^0 = \gamma x^0 - \gamma\beta x^1 \Rightarrow c\bar{t} = \gamma ct - \gamma\beta x$
$\bar{t} = \gamma t - \frac{\gamma v}{c^2} x$

Show that $\Lambda^\mu_\nu = \frac{\partial \bar{x}^\mu}{\partial x^\nu}$

In more advanced texts, you'll see

$\bar{x}^\mu = \frac{\partial \bar{x}^\mu}{\partial x^\nu} x^\nu$

Any 4-vector is contravariant if

$\bar{a}^\mu = \frac{\partial \bar{x}^\mu}{\partial x^\nu} a^\nu$

Any 4-vector is covariant if

$\bar{a}_\mu = \frac{\partial x^\mu}{\partial \bar{x}^\nu} a_\nu$

Find the covariant transform matrix for v is x-direction.

How can I change a covariant vector into a contravariant one.

Enter the metric $g_{\mu\nu}$ or $g^{\mu\nu}$

In special relativity $g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = g^{\mu\nu}$

$g_{\mu\nu} a^\nu = a_\mu \; ; \; g^{\mu\nu} a_\nu = a^\mu$

Inner products!

4-vector analog of $\vec{a} \cdot \vec{b}$ $\qquad \underbrace{a_\mu b^\mu}$

invariant in all reference.

Griffiths: $x_\mu x^\mu = I$ = same in all reference frames.
$d^2 - (ct)^2$

Title: Jul 30 - 6:39 PM (2 of 2)