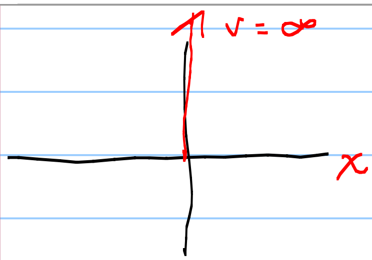
Review Session today at 4:00

$\beta = \frac{m\alpha}{\hbar^2 k}$

Note Title 2/24/2008



$$F + G = A + B$$
$$F - G = A(1+2i\beta) - B(1-2i\beta)$$

want $\begin{bmatrix} F \\ B \end{bmatrix}$ in terms of $\begin{bmatrix} A \\ G \end{bmatrix}$

out in

$$\boxed{\begin{bmatrix} F \\ B \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} A \\ G \end{bmatrix}}$$

$$F - B = A - G$$
$$F + (1-2i\beta)B = (1+2i\beta)A + G$$

LHS RHS

$$\begin{bmatrix} 1 & -1 \\ 1 & (1-2i\beta) \end{bmatrix} \begin{bmatrix} F \\ B \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ (1+2i\beta) & 1 \end{bmatrix} \begin{bmatrix} A \\ G \end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix} F \\ B \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & (1-2i\beta) \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1 \\ (1+2i\beta) & 1 \end{bmatrix} \begin{bmatrix} A \\ G \end{bmatrix}$$

$$\Downarrow$$

$$= \frac{1}{2-2i\beta} \begin{bmatrix} 1-2i\beta & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1+2i\beta & 1 \end{bmatrix} \begin{bmatrix} A \\ G \end{bmatrix}$$

$$= \frac{1}{2-2i\beta} \begin{bmatrix} 2 & 2i\beta \\ 2i\beta & 2 \end{bmatrix}$$

$$\boxed{S = \frac{1}{1-i\beta} \begin{bmatrix} 1 & i\beta \\ i\beta & 1 \end{bmatrix}}$$

$$\mathrm{Det}[S] = \underline{1}$$

$$|S_{11}|^2 = \frac{1}{1+\beta^2} = |S_{22}|^2$$

$$|S_{12}|^2 = \frac{\beta^2}{1+\beta^2} = |S_{21}|^2$$

S-matrix gives outgoing (B, F) in terms of incoming (A, G)

what about Right (A, B) in terms of left (F, G)?

$$\begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$

Eg. δ - potential

$$F + G = A + B$$

$$F - G = A(1+2i\beta) - B(1-2i\beta)$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} F \\ G \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ (1+2i\beta) & -(1-2i\beta) \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} F \\ G \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} F \\ G \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ (1+2i\beta) & -(1-2i\beta) \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1+i\beta & i\beta \\ -i\beta & 1-i\beta \end{bmatrix}}_{\text{transfer matrix}}\begin{bmatrix} A \\ B \end{bmatrix}$$

$$\mathrm{Det}(M) = 1$$

out $\begin{bmatrix} F \\ B \end{bmatrix} = S \begin{bmatrix} A \\ G \end{bmatrix}$ in

right $\begin{bmatrix} F \\ G \end{bmatrix} = M \begin{bmatrix} A \\ B \end{bmatrix}$ left

For HW you will show

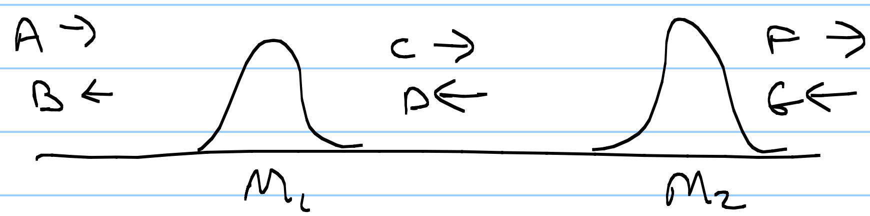
$$S = \frac{1}{M_{22}} \begin{pmatrix} -M_{21} & 1 \\ \det(M) & M_{12} \end{pmatrix}$$

$$M = \frac{1}{S_{12}} \begin{pmatrix} -\det(S) & S_{22} \\ -S_{11} & 1 \end{pmatrix}$$

So $R_L = |S_{11}|^2 = \left|\frac{M_{21}}{M_{22}}\right|^2 \quad T_L = |S_{21}|^2 = \left|\frac{\det(M)}{M_{22}}\right|^3$

$R_R = |S_{22}|^2 = \left|\frac{M_{12}}{M_{22}}\right|^2 \quad T_R = |S_{12}|^2 = \left|\frac{1}{M_{22}}\right|^2$

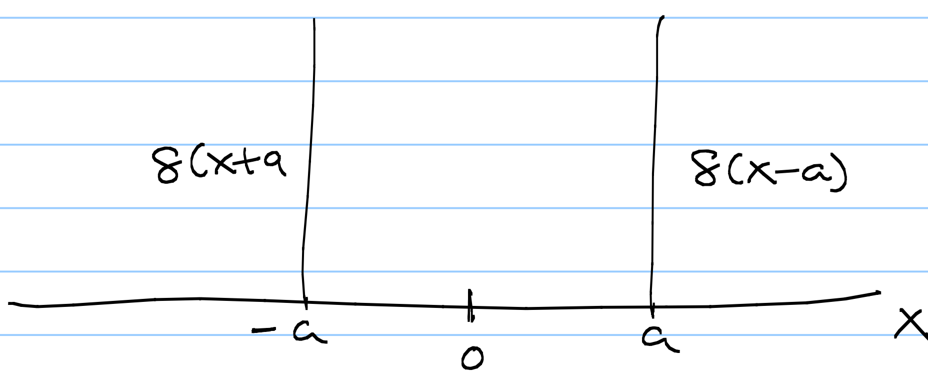
Now suppose you have 2 isolated potentials



$$\begin{pmatrix} F \\ G \end{pmatrix} = M_2 \begin{pmatrix} C \\ D \end{pmatrix} \qquad \begin{pmatrix} C \\ D \end{pmatrix} = M_1 \begin{bmatrix} A \\ B \end{bmatrix} \Rightarrow$$

$$\begin{pmatrix} F \\ G \end{pmatrix} = M_2 \, M_1 \begin{bmatrix} A \\ B \end{bmatrix}$$

How about 2 δ potentials



$M_1 \qquad M_2$

You can show (HW 2.53)

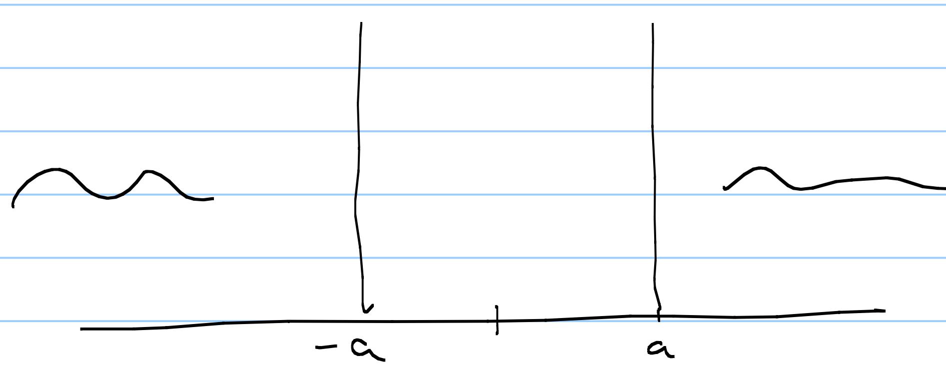
$$M_1 = \begin{pmatrix} 1+i\beta & i\beta e^{-2ika} \\ -i\beta e^{2ika} & 1-i\beta \end{pmatrix}$$

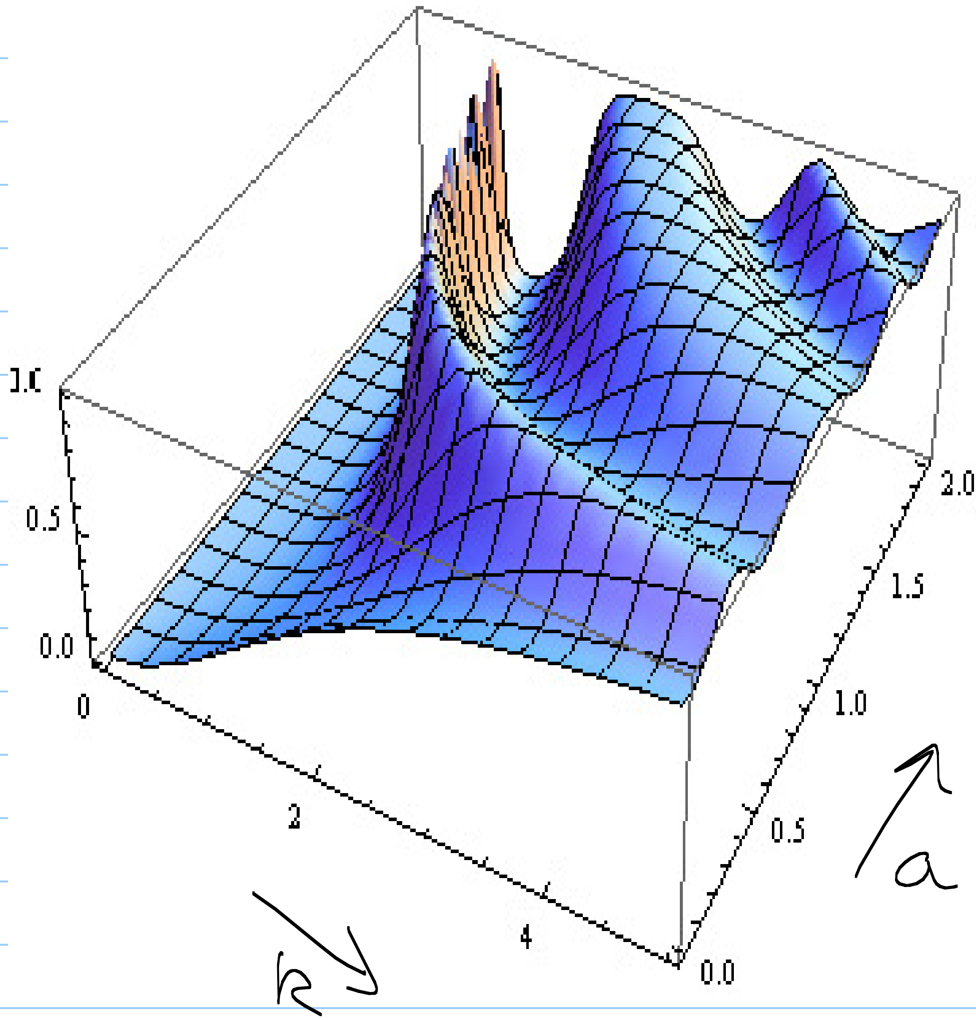
$$M_2 = \begin{pmatrix} 1+i\beta & i\beta e^{2ika} \\ -i\beta e^{-2ika} & 1-i\beta \end{pmatrix}$$

$$\begin{bmatrix} \left(1+2i\beta+\beta^2(4e^{ika}-1)\right) & \left(2i\beta\cos(2ka)+\beta\sin 2ka\right) \\ -2i\beta\left(\cos(2ka)+\beta\sin(2ka)\right) & \left(1-2i\beta+\beta^2(4e^{-4ika}-1)\right) \end{bmatrix}$$

$$T_L = T_R = T = \frac{1}{|M_{22}|^2}$$

$$T = \frac{1}{1+4\beta^2\left(\cos ka+\beta\sin(2ka)\right)^2}$$





As a reminder consider

1) T for a single $\delta(x)$

$$T = \frac{1}{1 + m\alpha^2/2\hbar E} \qquad \underline{2.141}$$

2) T for a square well

Fig 2.19 p. 82

Chapter 3.

2 key ingredients to Q.M.

1) wave functions
2) operators

Recall from 311

functions are abstract vectors

$\underbrace{f(x) + g(x)}_{\text{pointwise add.}}$ is like $\underbrace{\vec{x} + \vec{y}}_{\text{component-wise add.}}$

operators are represented by matrices in particular coordinates

$$(f, g) = \int_{-\infty}^{\infty} f^*(x)\, g(x)\, dx \quad \text{a "dot"}$$

product as

$$(\vec{x}, \vec{y}) = \vec{x}^T \vec{y} = \sum_{i=1}^{N} x_i y_i$$

New QM notation for vectors

$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} \to |\alpha\rangle$$

i.e., $\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}$ is the representation of $|\alpha\rangle$ in a particular basis.

But now inner products are complex

$$\boxed{\langle \alpha | \beta \rangle = \sum_{i=1}^{N} \alpha_i^* \beta_i}$$