MATH 225 - Differential Equations September 15, 2008

Project 1, Fall 2008 **Due**: September 24, 2008

Lab 1.3 - Logistics Population Models with Harvesting

For this project we consider lab 1.3 of Differential Equations pages 146 to 147. After reading this material construct a lab report addressing each of the following questions for cases 3, 4, 6, 8 of table 1.10 on page 147:[1]

1. Given a logistics growth model with constant harvesting,

$$\frac{dp}{dt} = kp\left(1 - \frac{p}{N}\right) - a, \quad k, N, a \in \mathbb{R}^+. \tag{1}$$

   (a) Construct a list of variables and parameters associated with (3) and describe the meaning of each.

   (b) Analytically solve (3) using the methods discussed in section 1.2 of the text.

   (c) Discuss qualitative behavior of the solutions to (3) through the equation's:

      i. Equilibrium Points

      ii. Phase Line

   (d) Using Euler's method and a slope field diagram address the following question:

      - For $a = a_1$, what will happen to the fish population for various initial conditions?

2. Given a logistics growth model with periodic harvesting,

$$\frac{dp}{dt} = kp\left(1 - \frac{p}{N}\right) - a(1 + \sin(bt)), \quad k, N, a, b \in \mathbb{R}^+. \tag{2}$$

   (a) In this case what do the parameters a and b represent?

   (b) Is it possible to solve (3) using the methods discussed in section 1.2 of the text?

   (c) Using Euler's method and a slope field diagram address the following questions:

      i. For $a = a_1$ and $b = 1$ what will happen to the fish population for various initial conditions?

      ii. For $a = a_2$ and $b = 1$ what will happen to the fish population for various initial conditions?

   Explain why there are no equilibrium points and thus no phase line for this problem.

3. Summary and conclusions. In a short essay format summarize your results from the previous questions. Compare and contrast each of the two models. Be sure to justify your conclusions by referencing your previous summary and analysis.

[1] Your report should be well organized and clearly presented. If steps are unclear then include more steps or make annotations clarifying the procedure and purpose. Be sure to label and title any included graphs or tables of data.

Lab Report - Logistics Population Models with Harvesting

In the following we respond to questions associated with Lab 1.3 of Differential Equations. This report is organized into three sections. In the first section we address a logistics growth model equation with constant harvesting through quantitative, qualitative and numerical analyses. A similar analysis is conducted on the logistics growth model with periodic harvesting in the second section. The third section will consist of a summary of results and report of conclusions associated with a comparison of these two models.

1. **Logistics Growth With Constant Harvesting**

$$\frac{dp}{dt} = f_1(p) = kp\left(1 - \frac{p}{N}\right) - a, \quad k, N, a \in \mathbb{R}^+. \tag{3}$$

(a) Variable and Parameter Listing:

i. $k \equiv$growth parameter : This parameter describes the rate of growth of the population p.

ii. $N \equiv$carrying capacity : This parameter describes the total amount of p that the resources can support.

iii. $a \equiv$harvesting rate : This parameter describes the rate that p will be taken from the system.

iv. $p \equiv$population : This dependent variable describes the population as a function of time.

v. $t \equiv$time : This independent variable parameterizes the evolution of the population, p.

(b) To solve this ODE analytically we note that (3) is autonomous and therefore separable. Applying separation of variables to (3) gives the following:

$$\begin{aligned}
\int \frac{dp}{kp\left(1 - \frac{p}{n}\right) - a} &= \int dt = t + C, \quad C \in \mathbb{R} && (4)\\
&= \int \left(\frac{A}{p - p_1} + \frac{B}{p - p_2}\right) dp && (5)\\
&= A \ln|p - p_1| + B \ln|p - p_2| && (6)\\
&= \ln|(p - p_1)^A (p - p_2)^B|, && (7)
\end{aligned}$$

which implies,

$$(p - p_1)^A (p - p_2)^B = Ce^t, \tag{8}$$

where p_1 and p_2 are roots to the quadratic polynomial in p and $A = (p_1 - p_2)^{-1}$, $B = (p_2 - p_1)^{-1}$ are found by partial fractions. If we assume the initial population $p(0) = p_0$ is given then we find that $C = (p_0 - p_1)^A (p_0 - p_2)^B$. It is not, in general, clear how we should solve for p explicitly. To do this we would need values for N, k, a to find p_1 and p_2 and thus A and B. If these numbers were known then polynomial root finding would give explicit formula for p.

(c) We now address the qualitative information that is given by the differential equation itself. To do this we first find the equilibrium solutions of (3) by solving,

$$\frac{dp}{dt} = 0 = kp\left(1 - \frac{p}{N}\right) - a \Rightarrow p^2 - Np + \frac{aN}{k} = 0, \tag{9}$$

to get the equilibrium solutions,

$$\begin{aligned}
p_1(t) &= \frac{N + \sqrt{N^2 - 4\frac{aN}{k}}}{2}, && (10)\\
p_2(t) &= \frac{N - \sqrt{N^2 - 4\frac{aN}{k}}}{2}, && (11)
\end{aligned}$$

assuming that $kN \neq 4a$. Otherwise, $p_1 = p_2$. We also note that p_2 is not physical for $\sqrt{N^2 - 4\frac{aN}{k}} > N$ or $N - 4\frac{a}{k} < 0$ and that for physically relevant cases $p_1(t) > p_2(t)$ for all t. To classify these equilibria we define $p_1 = p_+$ and $p_2 = p_-$ apply linearization to get,

$$\left.\frac{df}{dp}\right|_{p=p_\pm} = k\left(1 - \frac{2p_\pm}{N}\right) \tag{12}$$

$$= k\left(\pm\sqrt{1 - \frac{4a}{kN}}\right), \tag{13}$$

which implies that $p_1 = p_+$ is a sink and $p_2 = p_-$ is a source. Figure 1.1 shows the phase line for the system for $kN \neq 4a$ and the special case where $p_1 = p_2$.

(d) The following table correlates figures to parameter choices in table 1.10 page 147.

Figure Label	**Figure Type**	**Choice**
Fig. 1.2	Slope Fields	3
Fig. 1.3	Euler's Method	3
Fig. 1.4	Slope Fields	4
Fig. 1.5	Euler's Method	4
Fig. 1.6	Slope Fields	6
Fig. 1.7	Euler's Method	6
Fig. 1.8	Slope Fields	8
Fig. 1.9	Euler's Method	8

From these figures we can conclude that as we increase the harvesting parameter the trajectories change giving rise to decay for particular initial populations. This is due to the fact that as a increases the equilibrium solutions of the system get closer together. As these equilibria get closer together the source, p_2, moves up the p-axis and consequently decaying trajectories with initial populations between zero and p_2 are created. From this we can also conclude that as the two equilibria get closer and closer to each other the 'amount' of trajectories, which grow in time decreases. Thus it is possible to increase the harvesting parameter to a point where there are no initial populations, which grow in time. Moreover, it is possible harvest to a point where there are no initial populations whose trajectory is viable in the long-term. Using this model for a biological system gives insight into how much one could harvest without destroying the population in finite-time. This can be seen mathematically through the phase line in figure 1.1.

2. **Logistics Growth With Periodic Harvesting**

$$\frac{dp}{dt} = f_2(p,t) = kp\left(1 - \frac{p}{N}\right) - a(1 + \sin(bt)), \quad k, N, a, b \in \mathbb{R}^+. \tag{14}$$

(a) In this case we have that b is a parameter, which controls the frequency of the periodic harvesting and a represents the overall amplitude of the periodic harvesting. We note that in this case the harvesting can be as much as $2a$ and as little as 0.

(b) Noting $f_2(p,t) \neq h(p)g(t)$ implies that (14) is not separable. There is no clear way to solve this differential equation analytically.

(c) Though there are no clear analytic solutions we can still use qualitative and numerical techniques to the problem. The following table correlates figures to parameter choices in table 1.10 page 147 for $a = a_1$ and $b = 1$.

Figure Label	**Figure Type**	**Choice**
Fig. 1.10	Slope Fields	3
Fig. 1.11	Euler's Method	3
Fig. 1.12	Slope Fields	4
Fig. 1.13	Euler's Method	4
Fig. 1.14	Slope Fields	6
Fig. 1.15	Euler's Method	6
Fig. 1.16	Slope Fields	8
Fig. 1.17	Euler's Method	8

Figures 1.18 and 1.19 show slope fields and numerical approximations for $a = a_2$ and $b = 1$.

In general we notice similar behavior of the population as we did in part (1). Specifically, as we increase the amplitude of the harvesting parameter we create trajectories are shifted in the negative $y-$direction. However, in this case, since the system is non-autonomous there are no classical equilibrium solutions. Instead, for this system, those trajectories, which do not go extinct tend to a steady long-term oscillatory behavior.

3. **Conclusions and Summary**

In this lab we are presented two possible models for population harvesting. With these models one can conclude that there is a relationship between harvesting rate and the long-term population. We, in general, note that increased harvesting leads to 'more' extinction-trajectories. Moreover, it is possible to use the autonomous system to infer behaviors of the non-autonomous system. While the autonomous system has equilibrium solutions the non-autonomous seems to have what could be considered 'steady-state solutions'. These steady-state solution can be used, like equilibrium solutions, as reference points to describe the behavior of neighboring solutions.

While it is clear how the presence of constant harvesting can shift an autonomous system's equilibrium solution it is not obvious that this should be true of a non-autonomous system. If we compare Figure 1.12 and Figure 1.6 we see that for particular harvesting values both populations settle to a trajectory that remains viable in the long-term. In contrast to this we have Figures 1.8 and 1.16, which show that the effect of increased harvesting tends to lead to the loss of the population in finite-time. If we think of these long-term patterns as steady-state solution then we can conclude that the effect of harvesting is to shift these steady-state solutions, which creates initial populations destined for extinction similar to the autonomous system.

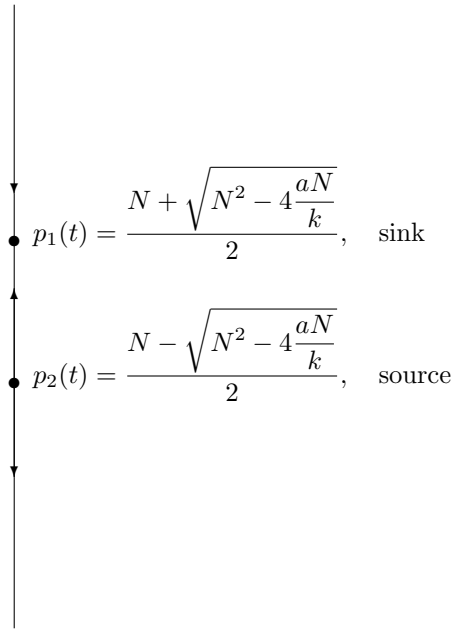
Equilibrium solutions as well as steady-state solutions make it possible to determine the long-term behavior of a particular solution satisfying an initial condition. This is important to the physical reality of the mathematical model, because one would naturally like to know how much harvesting can take place without decimating a particular population. Since it is unlikely that constant harvesting is possible for a particular population it makes sense to consider periodic harvesting instead.

While there are clearly qualitative similarities in the models there are some interesting quantitative differences. If we compare Figure 1.13 to Figure 1.5 we see that oscillatory harvesting has a detrimental effect causing the purple trajectory to go extinct more quickly than in the constant harvesting case. Clearly, sinusoidal harvesting, while more complicated, does not guarantee the long-term viability of all initial populations. This is because the harvesting does not take into account the current population at the time of harvest. A way to avoid this decay to extinction is to harvest the population at a rate proportional to the population itself. In this way the harvesting rate will decrease when the population is low and possibly might save a population from extinction. This method is outlined in problems 20 and 21 of chapter one section seven of the text and shows that this more complicated autonomous model gives better(safer) results when a population gets very low.

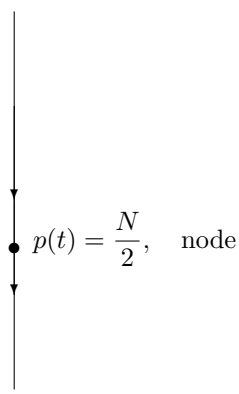
Figures

Figure 1.1

Slope Line for $kN \neq 4a$.



Slope Line for $kN = 4a$. [2]

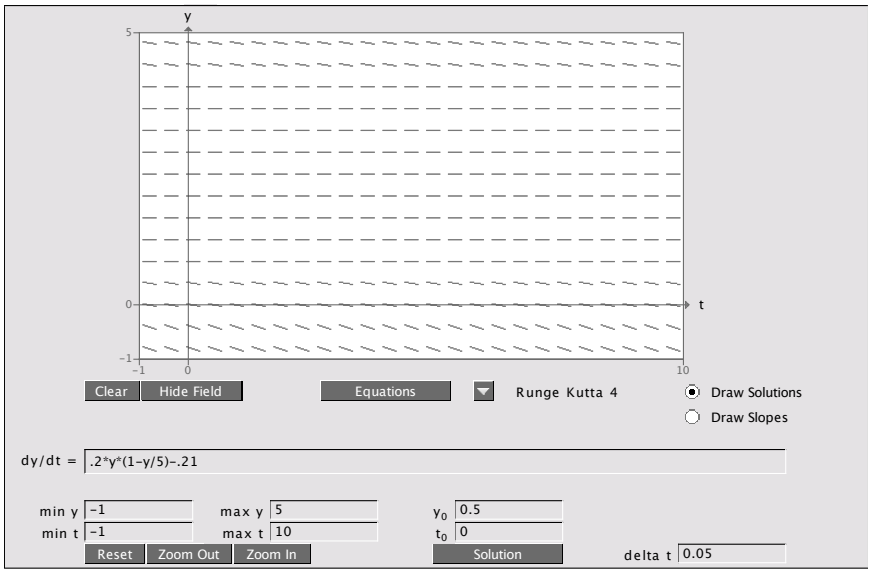


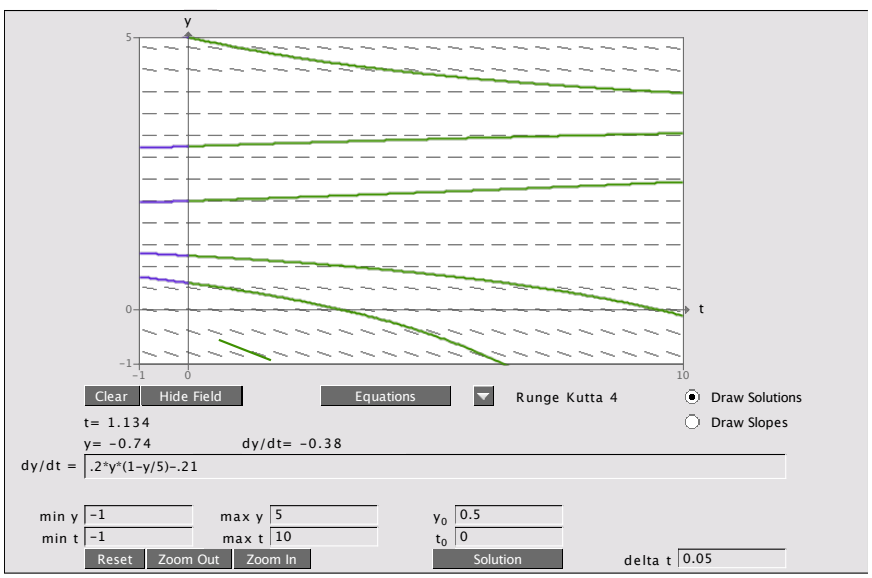
[2] In general, it is not obvious why this case yields a node. The best way to think about it is the following:

1. Think of $f_1(p)$ as a parabola that opens downward in the $y - f_1$ plane and has roots p_1 and p_2.
2. As a increases the parabola is shifted in the negative f_1 direction. This causes the roots to get closer together.
3. When $kN = 4a$ the roots join and become a double root and f_1 is negative on both sides of this root. This implies that the root (aka equilibrium solution) is a node. You may want to draw a picture of this to convince yourself.

It is interesting to note that this sort of change in the equilibria of an autonomous ODE with respect to a parameter is called a *bifurcation*, specifically a *saddle-node bifurcation*, and is highly important in the study of ODE's and their connections to mathematical chaos. We do not have time for this section, 1.7 of your text. It is accessible with the background we have and those interested should check it out.

Figure 1.2


y
5
0
-1
-1
0
10
t
Clear
Hide Field
Equations
Runge Kutta 4
Draw Solutions
Draw Slopes
dy/dt =
.2*y*(1-y/5)-.21
min y
-1
max y
5
y_0
0.5
min t
-1
max t
10
t_0
0
Reset
Zoom Out
Zoom In
Solution
delta t
0.05


y
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0
-1
-1
0
10
t
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Hide Field
Equations
Runge Kutta 4
Draw Solutions
Draw Slopes
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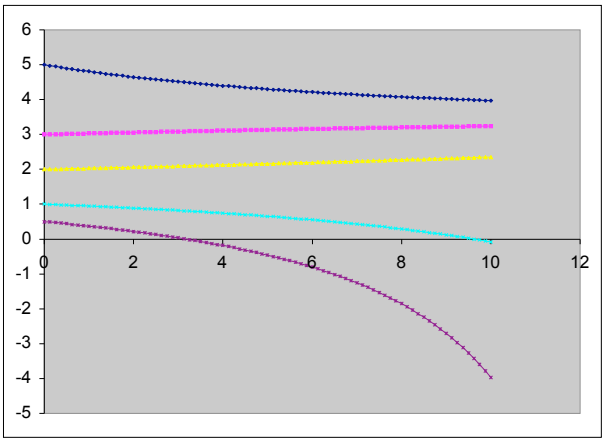
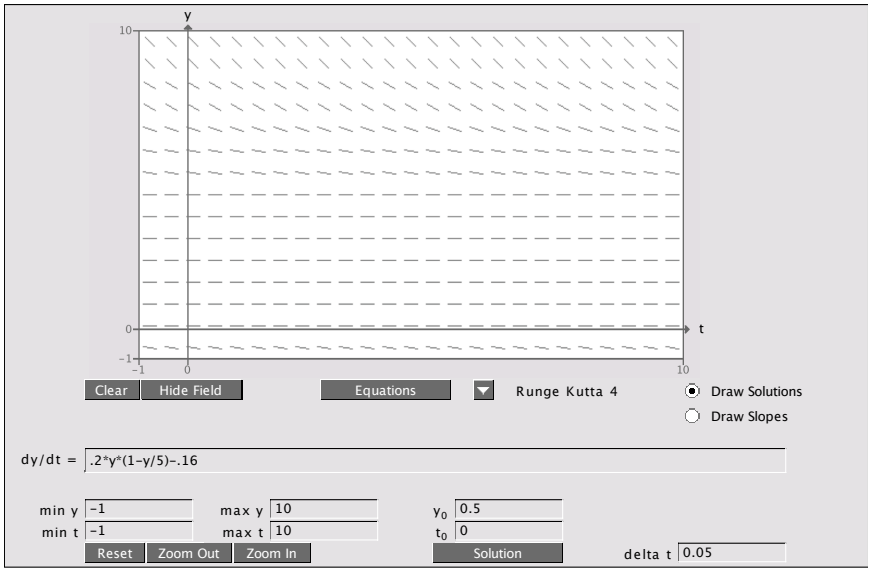
y= -0.74
dy/dt= -0.38
dy/dt =
.2*y*(1-y/5)-.21
min y
-1
max y
5
y_0
0.5
min t
-1
max t
10
t_0
0
Reset
Zoom Out
Zoom In
Solution
delta t
0.05

Figure 1.3



DELTA T	0.125														
k	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)
0	0	5	-0.21	0	3	0.03	0	2	0.03	0	1	-0.05	0	0.5	-0.12
1	0.125	4.97375	-0.20478	0.125	3.00375	0.029849	0.125	2.00375	0.030149	0.125	0.99375	-0.05075	0.125	0.485	-0.12241
2	0.25	4.9481528	-0.19974	0.25	3.007481	0.029699	0.25	2.007519	0.030298	0.25	0.987406	-0.05152	0.25	0.469699	-0.12488
3	0.375	4.92318554	-0.19487	0.375	3.011193	0.029547	0.375	2.011306	0.030447	0.375	0.980966	-0.0523	0.375	0.454088	-0.12743
4	0.5	4.8988264	-0.19017	0.5	3.014887	0.029396	0.5	2.015112	0.030595	0.5	0.974429	-0.05309	0.5	0.438159	-0.13005
5	0.625	4.87505456	-0.18564	0.625	3.018561	0.029244	0.625	2.018936	0.030743	0.625	0.967792	-0.05391	0.625	0.421904	-0.13274
6	0.75	4.85185014	-0.18125	0.75	3.022217	0.029092	0.75	2.022779	0.03089	0.75	0.961054	-0.05473	0.75	0.405311	-0.13551
7	0.875	4.82919415	-0.17701	0.875	3.025853	0.028939	0.875	2.02664	0.031037	0.875	0.954212	-0.05558	0.875	0.388373	-0.13836
8	1	4.80706842	-0.1729	1	3.029471	0.028786	1	2.03052	0.031184	1	0.947265	-0.05644	1	0.371078	-0.14129
9	1.125	4.7854556	-0.16893	1.125	3.033069	0.028633	1.125	2.034418	0.031329	1.125	0.94021	-0.05732	1.125	0.353416	-0.14431
10	1.25	4.76433906	-0.16509	1.25	3.036648	0.02848	1.25	2.038334	0.031475	1.25	0.933045	-0.05821	1.25	0.335377	-0.14742
11	1.375	4.7437029	-0.16137	1.375	3.040208	0.028327	1.375	2.042269	0.031619	1.375	0.925768	-0.05913	1.375	0.316949	-0.15063
12	1.5	4.72353189	-0.15776	1.5	3.043749	0.028173	1.5	2.046221	0.031763	1.5	0.918377	-0.06006	1.5	0.29812	-0.15393
13	1.625	4.70381142	-0.15427	1.625	3.047271	0.02802	1.625	2.050191	0.031907	1.625	0.91087	-0.06101	1.625	0.278879	-0.15734
14	1.75	4.68452749	-0.15089	1.75	3.050773	0.027866	1.75	2.05418	0.03205	1.75	0.903243	-0.06199	1.75	0.259212	-0.16085
15	1.875	4.66566669	-0.1476	1.875	3.054256	0.027712	1.875	2.058186	0.032192	1.875	0.895495	-0.06298	1.875	0.239107	-0.16447
16	2	4.64721613	-0.14442	2	3.05772	0.027558	2	2.06221	0.032334	2	0.887623	-0.06399	2	0.218548	-0.1682
17	2.125	4.62916345	-0.14133	2.125	3.061165	0.027404	2.125	2.066252	0.032474	2.125	0.879624	-0.06502	2.125	0.197523	-0.17206
18	2.25	4.61149676	-0.13834	2.25	3.064591	0.027249	2.25	2.070311	0.032615	2.25	0.871496	-0.06608	2.25	0.176016	-0.17604
19	2.375	4.59420467	-0.13543	2.375	3.067997	0.027095	2.375	2.074388	0.032754	2.375	0.863236	-0.06716	2.375	0.154012	-0.18015
20	2.5	4.5772762	-0.1326	2.5	3.071384	0.026941	2.5	2.078482	0.032893	2.5	0.854841	-0.06826	2.5	0.131493	-0.18439
21	2.625	4.56070082	-0.12986	2.625	3.074751	0.026786	2.625	2.082594	0.033031	2.625	0.846308	-0.06939	2.625	0.108444	-0.18878
22	2.75	4.54446838	-0.12719	2.75	3.0781	0.026632	2.75	2.086723	0.033168	2.75	0.837634	-0.07054	2.75	0.084847	-0.19332
23	2.875	4.52856913	-0.1246	2.875	3.081429	0.026478	2.875	2.090869	0.033304	2.875	0.828817	-0.07171	2.875	0.060682	-0.19801
24	3	4.51299366	-0.12209	3	3.084738	0.026323	3	2.095032	0.03344	3	0.819853	-0.07292	3	0.03593	-0.20287
25	3.125	4.49773295	-0.11964	3.125	3.088029	0.026169	3.125	2.099212	0.033575	3.125	0.810738	-0.07414	3.125	0.010572	-0.20789
26	3.25	4.48277826	-0.11726	3.25	3.0913	0.026015	3.25	2.103409	0.033709	3.25	0.80147	-0.0754	3.25	-0.01541	-0.21309
27	3.375	4.46812121	-0.11494	3.375	3.094552	0.02586	3.375	2.107622	0.033842	3.375	0.792045	-0.07668	3.375	-0.04205	-0.21848
28	3.5	4.45375371	-0.11269	3.5	3.097784	0.025706	3.5	2.111852	0.033974	3.5	0.78246	-0.078	3.5	-0.06936	-0.22406
29	3.625	4.43966794	-0.11049	3.625	3.100998	0.025552	3.625	2.116099	0.034105	3.625	0.77271	-0.07934	3.625	-0.09737	-0.22985
30	3.75	4.42585638	-0.10836	3.75	3.104192	0.025398	3.75	2.120362	0.034235	3.75	0.762793	-0.08072	3.75	-0.1261	-0.23586
31	3.875	4.41231177	-0.10628	3.875	3.107366	0.025244	3.875	2.124641	0.034364	3.875	0.752703	-0.08212	3.875	-0.15558	-0.24208
32	4	4.39902709	-0.10425	4	3.110522	0.025091	4	2.128937	0.034492	4	0.742438	-0.08356	4	-0.18584	-0.24855
33	4.125	4.38599557	-0.10228	4.125	3.113658	0.024937	4.125	2.133249	0.03462	4.125	0.731993	-0.08503	4.125	-0.21691	-0.25526
34	4.25	4.37321067	-0.10036	4.25	3.116775	0.024784	4.25	2.137576	0.034746	4.25	0.721363	-0.08654	4.25	-0.24882	-0.26224
35	4.375	4.36066608	-0.09848	4.375	3.119873	0.02463	4.375	2.141919	0.034871	4.375	0.710546	-0.08809	4.375	-0.2816	-0.26949
36	4.5	4.34835569	-0.09666	4.5	3.122952	0.024477	4.5	2.146278	0.034995	4.5	0.699535	-0.08967	4.5	-0.31529	-0.27703
37	4.625	4.33627359	-0.09488	4.625	3.126012	0.024324	4.625	2.150653	0.035118	4.625	0.688327	-0.09129	4.625	-0.34992	-0.28488
38	4.75	4.32441409	-0.09314	4.75	3.129052	0.024172	4.75	2.155042	0.03524	4.75	0.676916	-0.09295	4.75	-0.38553	-0.29305
39	4.875	4.31277166	-0.09145	4.875	3.132074	0.024019	4.875	2.159447	0.035361	4.875	0.665298	-0.09465	4.875	-0.42216	-0.30156
40	5	4.30134095	-0.08979	5	3.135076	0.023867	5	2.163867	0.035481	5	0.653467	-0.09639	5	-0.45985	-0.31043
41	5.125	4.2901168	-0.08818	5.125	3.138059	0.023715	5.125	2.168303	0.035599	5.125	0.641419	-0.09817	5.125	-0.49866	-0.31968
42	5.25	4.27909421	-0.08661	5.25	3.141024	0.023564	5.25	2.172752	0.035716	5.25	0.629147	-0.1	5.25	-0.53862	-0.32933
43	5.375	4.26826833	-0.08507	5.375	3.143969	0.023412	5.375	2.177217	0.035832	5.375	0.616646	-0.10188	5.375	-0.57978	-0.3394
44	5.5	4.25763447	-0.08357	5.5	3.146896	0.023261	5.5	2.181696	0.035947	5.5	0.603911	-0.10381	5.5	-0.62221	-0.34993
45	5.625	4.24718807	-0.08211	5.625	3.149803	0.02311	5.625	2.186189	0.036061	5.625	0.590936	-0.10578	5.625	-0.66595	-0.36093
46	5.75	4.23692474	-0.08068	5.75	3.152692	0.02296	5.75	2.190697	0.036173	5.75	0.577713	-0.10781	5.75	-0.71106	-0.37244
47	5.875	4.2268402	-0.07928	5.875	3.155562	0.02281	5.875	2.195219	0.036284	5.875	0.564237	-0.10989	5.875	-0.75762	-0.38448
48	6	4.21693032	-0.07791	6	3.158413	0.02266	6	2.199754	0.036394	6	0.550501	-0.11202	6	-0.80568	-0.3971
49	6.125	4.20719107	-0.07658	6.125	3.161246	0.02251	6.125	2.204304	0.036503	6.125	0.536498	-0.11421	6.125	-0.85532	-0.41033
50	6.25	4.19761856	-0.07528	6.25	3.16406	0.022361	6.25	2.208866	0.03661	6.25	0.522222	-0.11646	6.25	-0.90661	-0.4242
51	6.375	4.18820902	-0.074	6.375	3.166855	0.022212	6.375	2.213443	0.036715	6.375	0.507664	-0.11878	6.375	-0.95963	-0.43876
52	6.5	4.17895877	-0.07276	6.5	3.169631	0.022064	6.5	2.218032	0.03682	6.5	0.492817	-0.12115	6.5	-1.01448	-0.45406

Figure 1.4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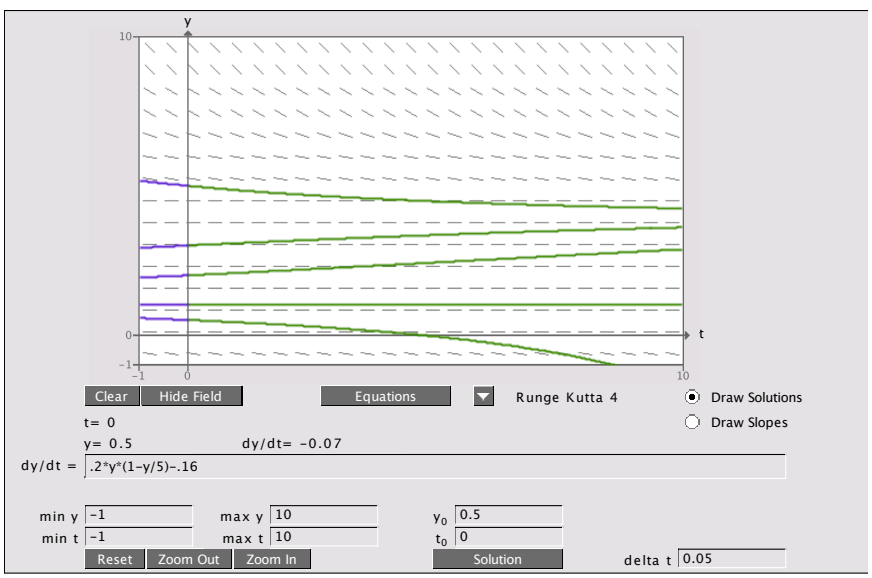
0
-1
-1
0
10
t
Clear
Hide Field
Equations
Runge Kutta 4
Draw Solutions
Draw Slopes
dy/dt =
.2*y*(1-y/5)-.16
min y
-1
max y
10
y_0
0.5
min t
-1
max t
10
t_0
0
Reset
Zoom Out
Zoom In
Solution
delta t
0.05


y
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0
-1
-1
0
10
t
Clear
Hide Field
Equations
Runge Kutta 4
Draw Solutions
Draw Slopes
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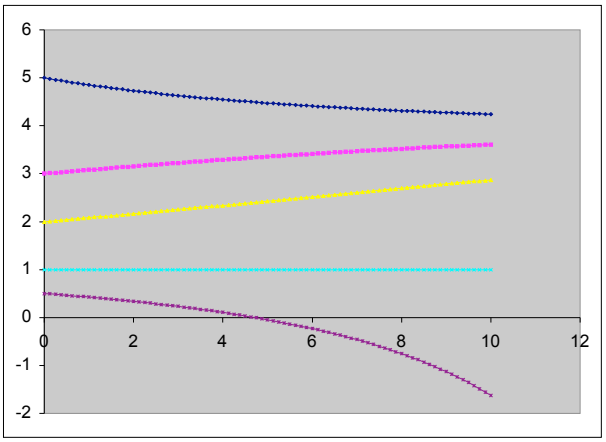
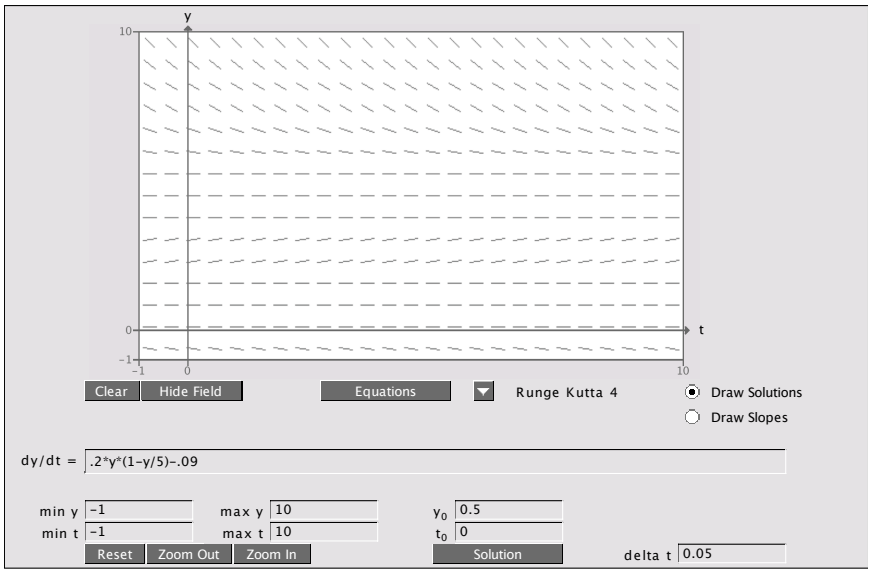
y= 0.5
dy/dt= -0.07
dy/dt =
.2*y*(1-y/5)-.16
min y
-1
max y
10
y_0
0.5
min t
-1
max t
10
t_0
0
Reset
Zoom Out
Zoom In
Solution
delta t
0.05

Figure 1.5



DELTA T	0.125														
k	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)
0	0	5	-0.16	0	3	0.08	0	2	0.08	0	1	0	0	0.5	-0.07
1	0.125	4.98	-0.15602	0.125	3.01	0.079596	0.125	2.01	0.080396	0.125	1	0	0.125	0.49125	-0.0714
2	0.25	4.960498	-0.15216	0.25	3.01995	0.079186	0.25	2.02005	0.080786	0.25	1	0	0.25	0.482325	-0.07284
3	0.375	4.94147775	-0.14843	0.375	3.029848	0.07877	0.375	2.030148	0.08117	0.375	1	0	0.375	0.47322	-0.07431
4	0.5	4.92292368	-0.14482	0.5	3.039694	0.078349	0.5	2.040294	0.081547	0.5	1	0	0.5	0.46393	-0.07582
5	0.625	4.90482088	-0.14133	0.625	3.049488	0.077923	0.625	2.050487	0.081918	0.625	1	0	0.625	0.454452	-0.07737
6	0.75	4.88715507	-0.13794	0.75	3.059228	0.077491	0.75	2.060727	0.082282	0.75	1	0	0.75	0.444781	-0.07896
7	0.875	4.86991252	-0.13466	0.875	3.068914	0.077053	0.875	2.071012	0.082639	0.875	1	0	0.875	0.434912	-0.08058
8	1	4.85308009	-0.13148	1	3.078546	0.076611	1	2.081342	0.082989	1	1	0	1	0.424839	-0.08225
9	1.125	4.83664516	-0.1284	1.125	3.088122	0.076164	1.125	2.091716	0.083332	1.125	1	0	1.125	0.414557	-0.08396
10	1.25	4.82059561	-0.12541	1.25	3.097643	0.075713	1.25	2.102132	0.083668	1.25	1	0	1.25	0.404062	-0.08572
11	1.375	4.80491979	-0.12251	1.375	3.107107	0.075257	1.375	2.112591	0.083997	1.375	1	0	1.375	0.393347	-0.08752
12	1.5	4.78960651	-0.11969	1.5	3.116514	0.074796	1.5	2.12309	0.084318	1.5	1	0	1.5	0.382407	-0.08937
13	1.625	4.77464502	-0.11696	1.625	3.125864	0.074332	1.625	2.13363	0.084631	1.625	1	0	1.625	0.371236	-0.09127
14	1.75	4.76002497	-0.11431	1.75	3.135155	0.073863	1.75	2.144209	0.084937	1.75	1	0	1.75	0.359828	-0.09321
15	1.875	4.74573641	-0.11173	1.875	3.144388	0.073391	1.875	2.154826	0.085234	1.875	1	0	1.875	0.348176	-0.09521
16	2	4.73176975	-0.10923	2	3.153562	0.072914	2	2.16548	0.085524	2	1	0	2	0.336274	-0.09727
17	2.125	4.71811577	-0.1068	2.125	3.162676	0.072434	2.125	2.176171	0.085805	2.125	1	0	2.125	0.324116	-0.09938
18	2.25	4.70476558	-0.10444	2.25	3.171731	0.071951	2.25	2.186896	0.086079	2.25	1	0	2.25	0.311693	-0.10155
19	2.375	4.69171062	-0.10214	2.375	3.180724	0.071465	2.375	2.197656	0.086344	2.375	1	0	2.375	0.299	-0.10378
20	2.5	4.67894265	-0.09991	2.5	3.189658	0.070975	2.5	2.208449	0.0866	2.5	1	0	2.5	0.286028	-0.10607
21	2.625	4.66645369	-0.09774	2.625	3.198529	0.070482	2.625	2.219274	0.086848	2.625	1	0	2.625	0.27277	-0.10842
22	2.75	4.65423608	-0.09563	2.75	3.20734	0.069987	2.75	2.23013	0.087087	2.75	1	0	2.75	0.259217	-0.11084
23	2.875	4.64228242	-0.09357	2.875	3.216088	0.069489	2.875	2.241016	0.087317	2.875	1	0	2.875	0.245361	-0.11334
24	3	4.63058555	-0.09158	3	3.224774	0.068988	3	2.251931	0.087538	3	1	0	3	0.231194	-0.1159
25	3.125	4.61913857	-0.08963	3.125	3.233398	0.068485	3.125	2.262873	0.087751	3.125	1	0	3.125	0.216707	-0.11854
26	3.25	4.60793483	-0.08774	3.25	3.241958	0.06798	3.25	2.273842	0.087954	3.25	1	0	3.25	0.20189	-0.12125
27	3.375	4.59696789	-0.08589	3.375	3.250456	0.067473	3.375	2.284836	0.088148	3.375	1	0	3.375	0.186733	-0.12405
28	3.5	4.58623152	-0.08409	3.5	3.25889	0.066963	3.5	2.295854	0.088333	3.5	1	0	3.5	0.171227	-0.12693
29	3.625	4.57571971	-0.08234	3.625	3.26726	0.066452	3.625	2.306896	0.088508	3.625	1	0	3.625	0.155361	-0.12989
30	3.75	4.56542664	-0.08064	3.75	3.275567	0.06594	3.75	2.31796	0.088674	3.75	1	0	3.75	0.139125	-0.13295
31	3.875	4.55534671	-0.07898	3.875	3.283809	0.065426	3.875	2.329044	0.088831	3.875	1	0	3.875	0.122506	-0.1361
32	4	4.54547446	-0.07736	4	3.291988	0.06491	4	2.340148	0.088978	4	1	0	4	0.105494	-0.13935
33	4.125	4.53580463	-0.07578	4.125	3.300101	0.064394	4.125	2.35127	0.089115	4.125	1	0	4.125	0.088075	-0.1427
34	4.25	4.52633213	-0.07424	4.25	3.30815	0.063876	4.25	2.362409	0.089243	4.25	1	0	4.25	0.070239	-0.14615
35	4.375	4.51705202	-0.07274	4.375	3.316135	0.063357	4.375	2.373565	0.089361	4.375	1	0	4.375	0.05197	-0.14971
36	4.5	4.50795952	-0.07128	4.5	3.324055	0.062837	4.5	2.384735	0.089469	4.5	1	0	4.5	0.033256	-0.15339
37	4.625	4.49905002	-0.06985	4.625	3.331909	0.062317	4.625	2.395918	0.089567	4.625	1	0	4.625	0.014081	-0.15719
38	4.75	4.49031901	-0.06845	4.75	3.339699	0.061796	4.75	2.407114	0.089655	4.75	1	0	4.75	-0.00557	-0.16111
39	4.875	4.48176216	-0.0671	4.875	3.347423	0.061275	4.875	2.418321	0.089733	4.875	1	0	4.875	-0.02571	-0.16517
40	5	4.47337526	-0.06577	5	3.355083	0.060753	5	2.429538	0.089801	5	1	0	5	-0.04635	-0.16936
41	5.125	4.46515421	-0.06447	5.125	3.362677	0.060232	5.125	2.440763	0.08986	5.125	1	0	5.125	-0.06752	-0.17369
42	5.25	4.45709505	-0.06321	5.25	3.370206	0.05971	5.25	2.451995	0.089908	5.25	1	0	5.25	-0.08923	-0.17817
43	5.375	4.44919395	-0.06197	5.375	3.37767	0.059188	5.375	2.463234	0.089946	5.375	1	0	5.375	-0.1115	-0.1828
44	5.5	4.44144716	-0.06077	5.5	3.385068	0.058666	5.5	2.474477	0.089974	5.5	1	0	5.5	-0.13435	-0.18759
45	5.625	4.43385108	-0.05959	5.625	3.392401	0.058145	5.625	2.485724	0.089992	5.625	1	0	5.625	-0.1578	-0.19256
46	5.75	4.42640218	-0.05844	5.75	3.399669	0.057624	5.75	2.496973	0.09	5.75	1	0	5.75	-0.18187	-0.1977
47	5.875	4.41909705	-0.05732	5.875	3.406872	0.057103	5.875	2.508223	0.089997	5.875	1	0	5.875	-0.20658	-0.20302
48	6	4.41193238	-0.05622	6	3.41401	0.056583	6	2.519472	0.089985	6	1	0	6	-0.23196	-0.20854
49	6.125	4.40490495	-0.05515	6.125	3.421083	0.056064	6.125	2.530721	0.089962	6.125	1	0	6.125	-0.25803	-0.21427
50	6.25	4.39801164	-0.0541	6.25	3.428091	0.055546	6.25	2.541966	0.08993	6.25	1	0	6.25	-0.28481	-0.22021
51	6.375	4.3912494	-0.05307	6.375	3.435035	0.055028	6.375	2.553207	0.089887	6.375	1	0	6.375	-0.31234	-0.22637
52	6.5	4.38461528	-0.05207	6.5	3.441913	0.054512	6.5	2.564443	0.089834	6.5	1	0	6.5	-0.34064	-0.23277

Figure 1.6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-1
0
10
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Hide Field
Equations
Runge Kutta 4
Draw Solutions
Draw Slopes
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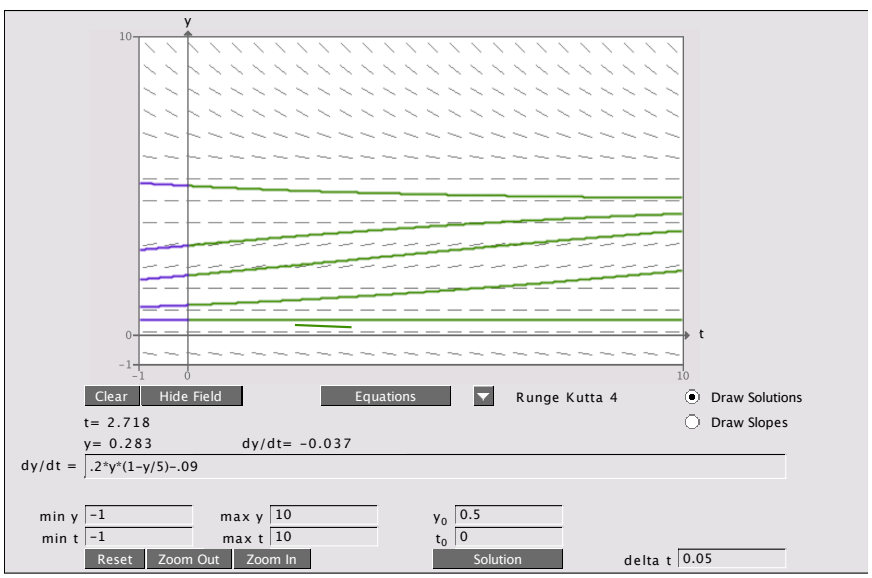
min y
-1
max y
10
y_0
0.5
min t
-1
max t
10
t_0
0
Reset
Zoom Out
Zoom In
Solution
delta t
0.05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-1
0
10
t
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Runge Kutta 4
Draw Solutions
Draw Slopes
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dy/dt =
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min y
-1
max y
10
y_0
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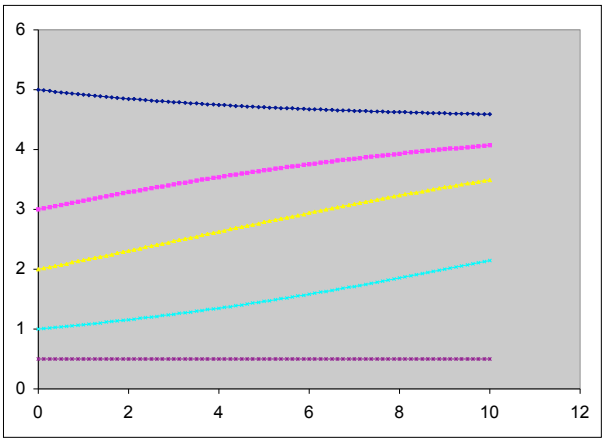
min t
-1
max t
10
t_0
0
Reset
Zoom Out
Zoom In
Solution
delta t
0.05

Figure 1.5



DELTA T	0.125														
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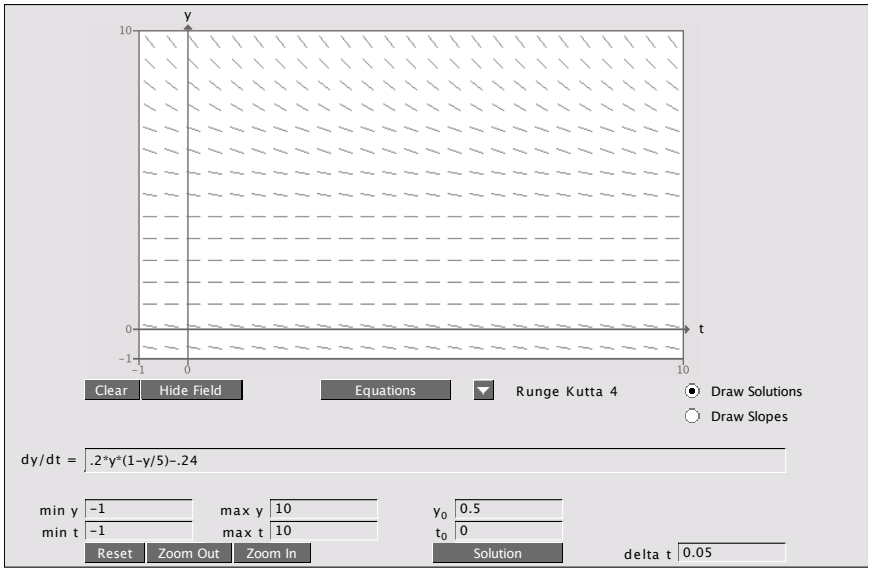
0	0	5	-0.09	0	3	0.15	0	2	0.15	0	1	0.07	0	0.5	0
1	0.125	4.98875	-0.08776	0.125	3.01875	0.149236	0.125	2.01875	0.150736	0.125	1.00875	0.071047	0.125	0.5	0
2	0.25	4.97778062	-0.08558	0.25	3.037404	0.148448	0.25	2.037592	0.151447	0.25	1.017631	0.072103	0.25	0.5	0
3	0.375	4.96708363	-0.08346	0.375	3.05596	0.147636	0.375	2.056523	0.152133	0.375	1.026644	0.073169	0.375	0.5	0
4	0.5	4.95665112	-0.08141	0.5	3.074415	0.146802	0.5	2.07554	0.152793	0.5	1.03579	0.074244	0.5	0.5	0
5	0.625	4.94647545	-0.07941	0.625	3.092765	0.145945	0.625	2.094639	0.153427	0.625	1.04507	0.075327	0.625	0.5	0
6	0.75	4.93654924	-0.07747	0.75	3.111008	0.145067	0.75	2.113817	0.154035	0.75	1.054486	0.07642	0.75	0.5	0
7	0.875	4.92686538	-0.07559	0.875	3.129142	0.144167	0.875	2.133071	0.154615	0.875	1.064039	0.077521	0.875	0.5	0
8	1	4.917417	-0.07376	1	3.147163	0.143247	1	2.152398	0.155167	1	1.073729	0.07863	1	0.5	0
9	1.125	4.90819748	-0.07198	1.125	3.165069	0.142307	1.125	2.171794	0.155691	1.125	1.083558	0.079748	1.125	0.5	0
10	1.25	4.8992004	-0.07025	1.25	3.182857	0.141348	1.25	2.191256	0.156187	1.25	1.093526	0.080873	1.25	0.5	0
11	1.375	4.89041959	-0.06856	1.375	3.200525	0.140371	1.375	2.210779	0.156654	1.375	1.103635	0.082007	1.375	0.5	0
12	1.5	4.88184906	-0.06693	1.5	3.218072	0.139375	1.5	2.230361	0.157092	1.5	1.113886	0.083148	1.5	0.5	0
13	1.625	4.87348303	-0.06534	1.625	3.235494	0.138362	1.625	2.249997	0.1575	1.625	1.124279	0.084296	1.625	0.5	0
14	1.75	4.86531593	-0.06379	1.75	3.252789	0.137332	1.75	2.269685	0.157878	1.75	1.134816	0.085451	1.75	0.5	0
15	1.875	4.85734233	-0.06228	1.875	3.269955	0.136287	1.875	2.289419	0.158226	1.875	1.145498	0.086613	1.875	0.5	0
16	2	4.84955701	-0.06082	2	3.286991	0.135226	2	2.309198	0.158544	2	1.156324	0.087781	2	0.5	0
17	2.125	4.84195492	-0.05939	2.125	3.303895	0.13415	2.125	2.329016	0.158831	2.125	1.167297	0.088956	2.125	0.5	0
18	2.25	4.83453116	-0.058	2.25	3.320663	0.13306	2.25	2.348869	0.159086	2.25	1.178417	0.090137	2.25	0.5	0
19	2.375	4.82728098	-0.05665	2.375	3.337296	0.131957	2.375	2.368755	0.159311	2.375	1.189684	0.091323	2.375	0.5	0
20	2.5	4.8201998	-0.05533	2.5	3.353791	0.130842	2.5	2.388669	0.159504	2.5	1.201099	0.092514	2.5	0.5	0
21	2.625	4.81328316	-0.05405	2.625	3.370146	0.129714	2.625	2.408607	0.159666	2.625	1.212663	0.093711	2.625	0.5	0
22	2.75	4.80652677	-0.0528	2.75	3.38636	0.128575	2.75	2.428565	0.159796	2.75	1.224377	0.094911	2.75	0.5	0
23	2.875	4.79992644	-0.05159	2.875	3.402432	0.127425	2.875	2.44854	0.159894	2.875	1.236241	0.096117	2.875	0.5	0
24	3	4.79347813	-0.0504	3	3.41836	0.126265	3	2.468527	0.15996	3	1.248256	0.097325	3	0.5	0
25	3.125	4.78717792	-0.04925	3.125	3.434143	0.125095	3.125	2.488522	0.159995	3.125	1.260421	0.098538	3.125	0.5	0
26	3.25	4.78102201	-0.04812	3.25	3.44978	0.123917	3.25	2.508521	0.159997	3.25	1.272738	0.099753	3.25	0.5	0
27	3.375	4.7750067	-0.04703	3.375	3.465269	0.12273	3.375	2.528521	0.159967	3.375	1.285208	0.100971	3.375	0.5	0
28	3.5	4.76912842	-0.04596	3.5	3.480611	0.121536	3.5	2.548517	0.159906	3.5	1.297829	0.102191	3.5	0.5	0
29	3.625	4.7633837	-0.04492	3.625	3.495803	0.120335	3.625	2.568505	0.159812	3.625	1.310603	0.103413	3.625	0.5	0
30	3.75	4.75776917	-0.0439	3.75	3.510845	0.119128	3.75	2.588481	0.159687	3.75	1.32353	0.104637	3.75	0.5	0
31	3.875	4.75228157	-0.04291	3.875	3.525736	0.117915	3.875	2.608442	0.15953	3.875	1.336609	0.105861	3.875	0.5	0
32	4	4.7469177	-0.04195	4	3.540475	0.116696	4	2.628383	0.159341	4	1.349842	0.107085	4	0.5	0
33	4.125	4.74167451	-0.041	4.125	3.555062	0.115474	4.125	2.648301	0.15912	4.125	1.363227	0.10831	4.125	0.5	0
34	4.25	4.73654899	-0.04009	4.25	3.569496	0.114247	4.25	2.668191	0.158868	4.25	1.376766	0.109534	4.25	0.5	0
35	4.375	4.73153823	-0.03919	4.375	3.583777	0.113017	4.375	2.68805	0.158585	4.375	1.390458	0.110757	4.375	0.5	0
36	4.5	4.72663941	-0.03832	4.5	3.597904	0.111784	4.5	2.707873	0.158272	4.5	1.404303	0.111978	4.5	0.5	0
37	4.625	4.7218498	-0.03746	4.625	3.611877	0.110549	4.625	2.727657	0.157927	4.625	1.4183	0.113197	4.625	0.5	0
38	4.75	4.71716672	-0.03663	4.75	3.625696	0.109312	4.75	2.747398	0.157552	4.75	1.432449	0.114413	4.75	0.5	0
39	4.875	4.71258758	-0.03582	4.875	3.63936	0.108074	4.875	2.767092	0.157146	4.875	1.446751	0.115627	4.875	0.5	0
40	5	4.70810986	-0.03503	5	3.652869	0.106836	5	2.786735	0.156711	5	1.461204	0.116836	5	0.5	0
41	5.125	4.70373111	-0.03426	5.125	3.666224	0.105597	5.125	2.806324	0.156247	5.125	1.475809	0.118041	5.125	0.5	0
42	5.25	4.69944896	-0.0335	5.25	3.679423	0.104358	5.25	2.825855	0.155753	5.25	1.490564	0.119242	5.25	0.5	0
43	5.375	4.69526108	-0.03277	5.375	3.692468	0.103121	5.375	2.845324	0.15523	5.375	1.505469	0.120436	5.375	0.5	0
44	5.5	4.69116522	-0.03205	5.5	3.705358	0.101884	5.5	2.864727	0.154679	5.5	1.520524	0.121625	5.5	0.5	0
45	5.625	4.6871592	-0.03135	5.625	3.718094	0.10065	5.625	2.884062	0.1541	5.625	1.535727	0.122807	5.625	0.5	0
46	5.75	4.68324087	-0.03066	5.75	3.730675	0.099418	5.75	2.903325	0.153493	5.75	1.551078	0.123982	5.75	0.5	0
47	5.875	4.67940817	-0.02999	5.875	3.743102	0.098188	5.875	2.922511	0.152859	5.875	1.566576	0.125149	5.875	0.5	0
48	6	4.67565907	-0.02934	6	3.755376	0.096961	6	2.941619	0.152199	6	1.582219	0.126307	6	0.5	0
49	6.125	4.6719916	-0.0287	6.125	3.767496	0.095738	6.125	2.960644	0.151512	6.125	1.598008	0.127456	6.125	0.5	0
50	6.25	4.66840387	-0.02808	6.25	3.779463	0.094519	6.25	2.979583	0.1508	6.25	1.61394	0.128596	6.25	0.5	0
51	6.375	4.66489399	-0.02747	6.375	3.791278	0.093304	6.375	2.998433	0.150063	6.375	1.630014	0.129725	6.375	0.5	0
52	6.5	4.66146016	-0.02688	6.5	3.802941	0.092094	6.5	3.017191	0.149301	6.5	1.64623	0.130843	6.5	0.5	0

Figure 1.8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-1
0
10
t
Clear
Hide Field
Equations
Runge Kutta 4
Draw Solutions
Draw Slopes
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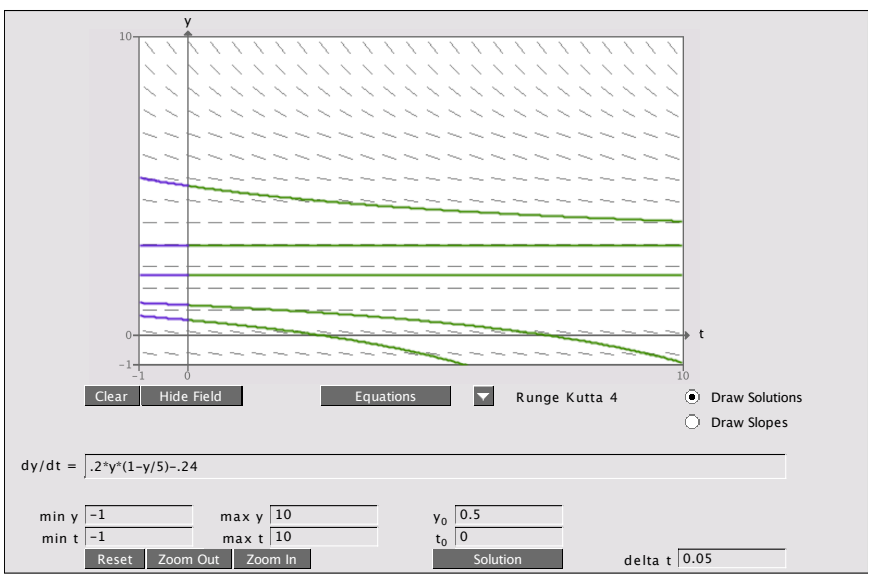
min y
-1
max y
10
y_0
0.5
min t
-1
max t
10
t_0
0
Reset
Zoom Out
Zoom In
Solution
delta t
0.05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Hide Field
Equations
Runge Kutta 4
Draw Solutions
Draw Slopes
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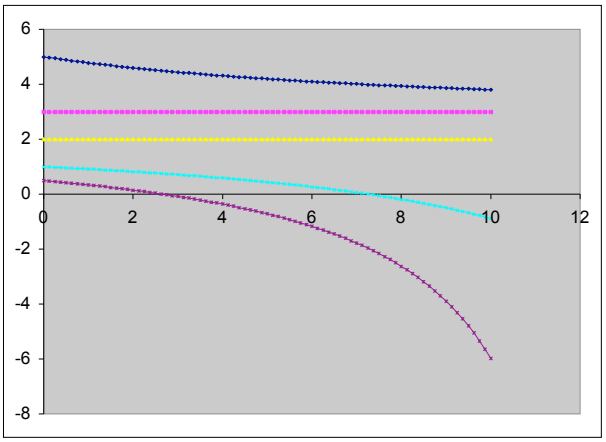
.2*y*(1-y/5)-.24
min y
-1
max y
10
y_0
0.5
min t
-1
max t
10
t_0
0
Reset
Zoom Out
Zoom In
Solution
delta t
0.05

Figure 1.9



DELTA T	0.125															
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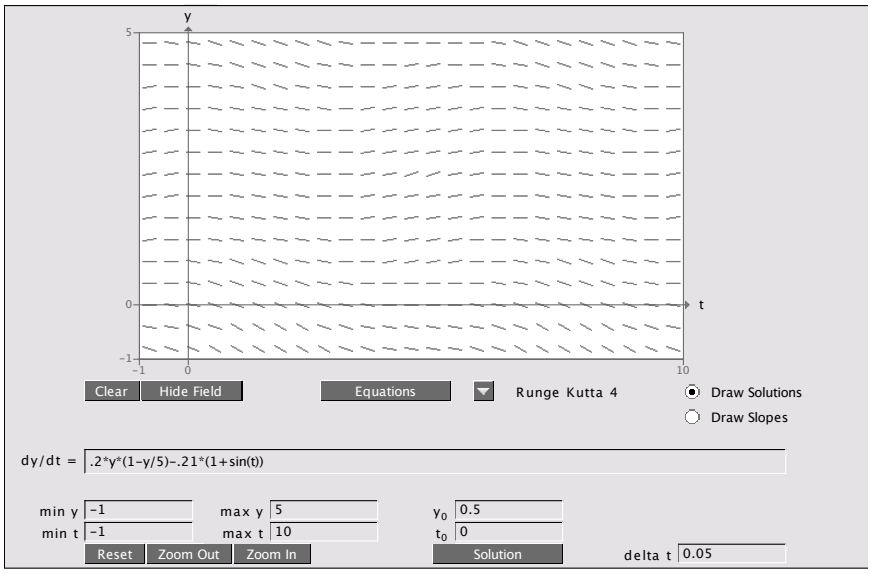
0	0	5	-0.24	0	3	0	0	2	0	0	1	-0.08	0	0.5	-0.15
1	0.125	4.97	-0.23404	0.125	3	0	0.125	2	0	0.125	0.99	-0.0812	0.125	0.48125	-0.15301
2	0.25	4.9407455	-0.22829	0.25	3	0	0.25	2	0	0.25	0.97985	-0.08243	0.25	0.462123	-0.15612
3	0.375	4.91220931	-0.22275	0.375	3	0	0.375	2	0	0.375	0.969545	-0.08369	0.375	0.442609	-0.15931
4	0.5	4.88436554	-0.21741	0.5	3	0	0.5	2	0	0.5	0.959084	-0.08498	0.5	0.422694	-0.16261
5	0.625	4.85718954	-0.21225	0.625	3	0	0.625	2	0	0.625	0.948462	-0.08629	0.625	0.402368	-0.166
6	0.75	4.83065783	-0.20728	0.75	3	0	0.75	2	0	0.75	0.937675	-0.08763	0.75	0.381618	-0.1695
7	0.875	4.804748	-0.20247	0.875	3	0	0.875	2	0	0.875	0.926721	-0.08901	0.875	0.36043	-0.17311
8	1	4.77943868	-0.19783	1	3	0	1	2	0	1	0.915595	-0.09041	1	0.338791	-0.17683
9	1.125	4.75470948	-0.19335	1.125	3	0	1.125	2	0	1.125	0.904293	-0.09185	1.125	0.316687	-0.18067
10	1.25	4.73054091	-0.18901	1.25	3	0	1.25	2	0	1.25	0.892812	-0.09332	1.25	0.294103	-0.18464
11	1.375	4.70691434	-0.18482	1.375	3	0	1.375	2	0	1.375	0.881147	-0.09483	1.375	0.271023	-0.18873
12	1.5	4.68381199	-0.18076	1.5	3	0	1.5	2	0	1.5	0.869293	-0.09637	1.5	0.247431	-0.19296
13	1.625	4.66121681	-0.17683	1.625	3	0	1.625	2	0	1.625	0.857247	-0.09795	1.625	0.223311	-0.19733
14	1.75	4.63911252	-0.17303	1.75	3	0	1.75	2	0	1.75	0.845004	-0.09956	1.75	0.198645	-0.20185
15	1.875	4.61748351	-0.16935	1.875	3	0	1.875	2	0	1.875	0.832559	-0.10121	1.875	0.173413	-0.20652
16	2	4.59631483	-0.16578	2	3	0	2	2	0	2	0.819907	-0.10291	2	0.147598	-0.21135
17	2.125	4.57559215	-0.16232	2.125	3	0	2.125	2	0	2.125	0.807044	-0.10464	2.125	0.121179	-0.21635
18	2.25	4.55530174	-0.15897	2.25	3	0	2.25	2	0	2.25	0.793963	-0.10642	2.25	0.094135	-0.22153
19	2.375	4.53543041	-0.15572	2.375	3	0	2.375	2	0	2.375	0.78066	-0.10825	2.375	0.066445	-0.22689
20	2.5	4.51596553	-0.15256	2.5	3	0	2.5	2	0	2.5	0.76713	-0.11011	2.5	0.038084	-0.23244
21	2.625	4.49689494	-0.1495	2.625	3	0	2.625	2	0	2.625	0.753365	-0.11203	2.625	0.009028	-0.2382
22	2.75	4.47820699	-0.14653	2.75	3	0	2.75	2	0	2.75	0.739362	-0.11399	2.75	-0.02075	-0.24417
23	2.875	4.45989048	-0.14365	2.875	3	0	2.875	2	0	2.875	0.725112	-0.11601	2.875	-0.05127	-0.25036
24	3	4.44193463	-0.14084	3	3	0	3	2	0	3	0.710611	-0.11808	3	-0.08256	-0.25679
25	3.125	4.42432907	-0.13812	3.125	3	0	3.125	2	0	3.125	0.695852	-0.1202	3.125	-0.11466	-0.26346
26	3.25	4.40706386	-0.13548	3.25	3	0	3.25	2	0	3.25	0.680827	-0.12238	3.25	-0.14759	-0.27039
27	3.375	4.3901294	-0.1329	3.375	3	0	3.375	2	0	3.375	0.66553	-0.12461	3.375	-0.18139	-0.27759
28	3.5	4.37351645	-0.1304	3.5	3	0	3.5	2	0	3.5	0.649954	-0.12691	3.5	-0.21609	-0.28509
29	3.625	4.35721613	-0.12797	3.625	3	0	3.625	2	0	3.625	0.63409	-0.12926	3.625	-0.25173	-0.29288
30	3.75	4.34121988	-0.1256	3.75	3	0	3.75	2	0	3.75	0.617932	-0.13169	3.75	-0.28834	-0.30099
31	3.875	4.32551942	-0.1233	3.875	3	0	3.875	2	0	3.875	0.601471	-0.13418	3.875	-0.32596	-0.30944
32	4	4.31010682	-0.12106	4	3	0	4	2	0	4	0.584699	-0.13674	4	-0.36464	-0.31825
33	4.125	4.29497438	-0.11888	4.125	3	0	4.125	2	0	4.125	0.567607	-0.13937	4.125	-0.40442	-0.32743
34	4.25	4.28011472	-0.11675	4.25	3	0	4.25	2	0	4.25	0.550187	-0.14207	4.25	-0.44535	-0.337
35	4.375	4.26552068	-0.11468	4.375	3	0	4.375	2	0	4.375	0.532428	-0.14485	4.375	-0.48747	-0.347
36	4.5	4.25118536	-0.11267	4.5	3	0	4.5	2	0	4.5	0.514321	-0.14772	4.5	-0.53085	-0.35744
37	4.625	4.23710211	-0.1107	4.625	3	0	4.625	2	0	4.625	0.495857	-0.15066	4.625	-0.57553	-0.36836
38	4.75	4.22326449	-0.10879	4.75	3	0	4.75	2	0	4.75	0.477024	-0.1537	4.75	-0.62157	-0.37977
39	4.875	4.20966629	-0.10692	4.875	3	0	4.875	2	0	4.875	0.457811	-0.15682	4.875	-0.66905	-0.39171
40	5	4.19630149	-0.1051	5	3	0	5	2	0	5	0.438209	-0.16004	5	-0.71801	-0.40422
41	5.125	4.1831643	-0.10332	5.125	3	0	5.125	2	0	5.125	0.418204	-0.16336	5.125	-0.76854	-0.41733
42	5.25	4.17024909	-0.10159	5.25	3	0	5.25	2	0	5.25	0.397784	-0.16677	5.25	-0.8207	-0.43108
43	5.375	4.15755043	-0.0999	5.375	3	0	5.375	2	0	5.375	0.376938	-0.1703	5.375	-0.87459	-0.44551
44	5.5	4.14506306	-0.09825	5.5	3	0	5.5	2	0	5.5	0.355651	-0.17393	5.5	-0.93028	-0.46067
45	5.625	4.1327819	-0.09664	5.625	3	0	5.625	2	0	5.625	0.33391	-0.17768	5.625	-0.98786	-0.47661
46	5.75	4.12070202	-0.09507	5.75	3	0	5.75	2	0	5.75	0.3117	-0.18155	5.75	-1.04744	-0.49337
47	5.875	4.10881864	-0.09353	5.875	3	0	5.875	2	0	5.875	0.289007	-0.18554	5.875	-1.10911	-0.51103
48	6	4.09712715	-0.09203	6	3	0	6	2	0	6	0.265814	-0.18966	6	-1.17299	-0.52963
49	6.125	4.08562308	-0.09057	6.125	3	0	6.125	2	0	6.125	0.242106	-0.19392	6.125	-1.23919	-0.54926
50	6.25	4.07430208	-0.08914	6.25	3	0	6.25	2	0	6.25	0.217866	-0.19833	6.25	-1.30785	-0.56999
51	6.375	4.06315994	-0.08774	6.375	3	0	6.375	2	0	6.375	0.193075	-0.20288	6.375	-1.3791	-0.5919
52	6.5	4.0521926	-0.08637	6.5	3	0	6.5	2	0	6.5	0.167716	-0.20758	6.5	-1.45309	-0.61508

Figure 1.10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Hide Field
Equations
Runge Kutta 4
Draw Solutions
Draw Slopes
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min y
-1
max y
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y_0
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min t
-1
max t
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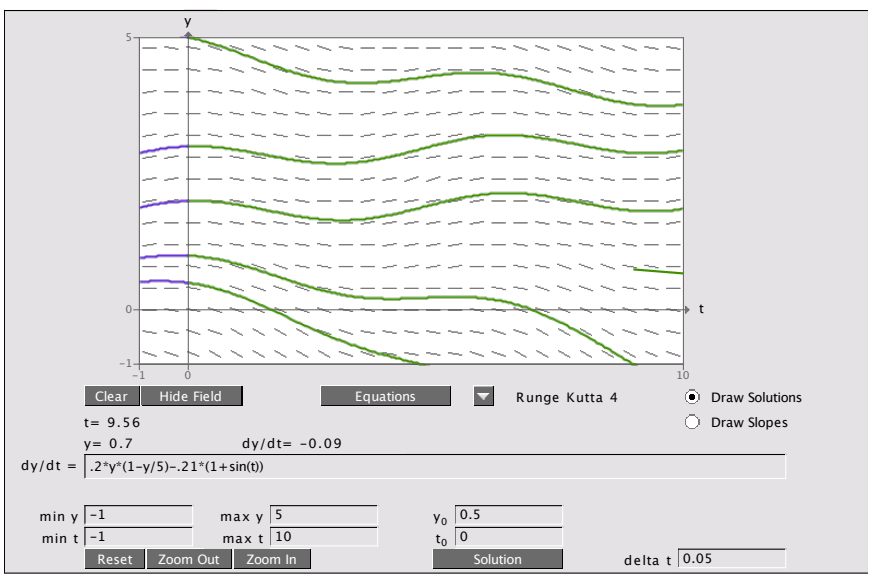
t_0
0
Reset
Zoom Out
Zoom In
Solution
delta t
0.05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t
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Equations
Runge Kutta 4
Draw Solutions
Draw Slopes
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dy/dt= -0.09
dy/dt =
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min y
-1
max y
5
y_0
0.5
min t
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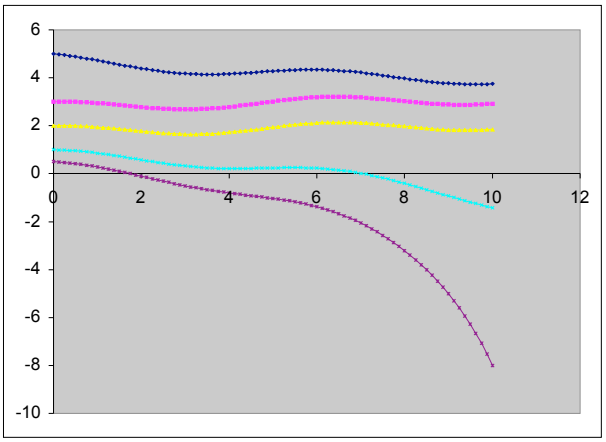
max t
10
t_0
0
Reset
Zoom Out
Zoom In
Solution
delta t
0.05

Figure 1.11



DELTA T	0.125														
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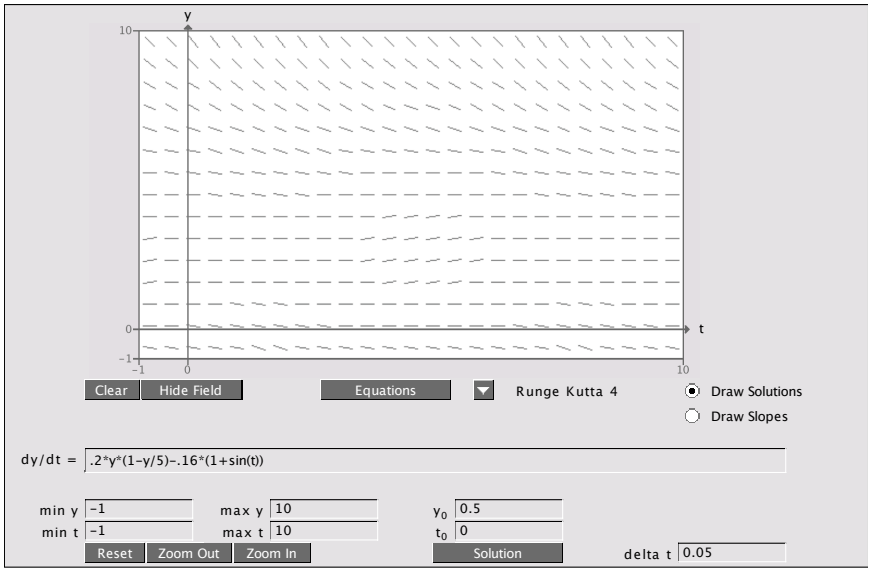
0	0	5	-0.21	0	3	0.03	0	2	0.03	0	1	-0.05	0	0.5	-0.12
1	0.125	4.97375	-0.23096	0.125	3.00375	0.003668	0.125	2.00375	0.003968	0.125	0.99375	-0.07693	0.125	0.485	-0.14859
2	0.25	4.94488009	-0.25105	0.25	3.004208	-0.02212	0.25	2.004246	-0.02179	0.25	0.984133	-0.10387	0.25	0.466426	-0.17737
3	0.375	4.91349855	-0.26992	0.375	3.001443	-0.04698	0.375	2.001523	-0.04686	0.375	0.97115	-0.13041	0.375	0.444255	-0.20596
4	0.5	4.87975902	-0.28721	0.5	2.995571	-0.0705	0.5	1.995666	-0.07085	0.5	0.954848	-0.15618	0.5	0.41851	-0.23398
5	0.625	4.84385783	-0.30262	0.625	2.986758	-0.09235	0.625	1.986809	-0.09341	0.625	0.935326	-0.1808	0.625	0.389262	-0.26108
6	0.75	4.80603068	-0.31586	0.75	2.975215	-0.11218	0.75	1.975133	-0.11416	0.75	0.912726	-0.20392	0.75	0.356627	-0.28691
7	0.875	4.76654877	-0.32667	0.875	2.961193	-0.12969	0.875	1.960863	-0.13281	0.875	0.887236	-0.22522	0.875	0.320764	-0.31115
8	1	4.72571454	-0.33486	1	2.944981	-0.14463	1	1.944262	-0.14906	1	0.859083	-0.24441	1	0.28187	-0.33351
9	1.125	4.6838569	-0.34025	1.125	2.926902	-0.15677	1.125	1.925629	-0.16267	1.125	0.828531	-0.26123	1.125	0.240181	-0.35375
10	1.25	4.64132622	-0.3427	1.25	2.907307	-0.16592	1.25	1.905295	-0.17343	1.25	0.795877	-0.27545	1.25	0.195963	-0.37163
11	1.375	4.59848899	-0.34213	1.375	2.886566	-0.17196	1.375	1.883615	-0.18118	1.375	0.761446	-0.28689	1.375	0.149509	-0.38698
12	1.5	4.55572226	-0.33851	1.5	2.865071	-0.17481	1.5	1.860967	-0.18581	1.5	0.725585	-0.29542	1.5	0.101136	-0.39966
13	1.625	4.51340805	-0.33184	1.625	2.84322	-0.1744	1.625	1.837741	-0.18724	1.625	0.688658	-0.30093	1.625	0.051179	-0.40956
14	1.75	4.47192754	-0.32218	1.75	2.82142	-0.17077	1.75	1.814337	-0.18544	1.75	0.651042	-0.30338	1.75	-1.6E-05	-0.41664
15	1.875	4.43165542	-0.30961	1.875	2.800073	-0.16396	1.875	1.791157	-0.18046	1.875	0.613119	-0.30277	1.875	-0.0521	-0.42089
16	2	4.3929542	-0.29428	2	2.779578	-0.15408	2	1.7686	-0.17235	2	0.575273	-0.29914	2	-0.10471	-0.42233
17	2.125	4.35616877	-0.27638	2.125	2.760319	-0.14128	2.125	1.747056	-0.16124	2.125	0.537881	-0.29256	2.125	-0.1575	-0.42106
18	2.25	4.32162106	-0.25613	2.25	2.742659	-0.12575	2.25	1.7269	-0.1473	2.25	0.50131	-0.28319	2.25	-0.21013	-0.41719
19	2.375	4.28960512	-0.23378	2.375	2.72694	-0.10773	2.375	1.708487	-0.13073	2.375	0.465912	-0.27117	2.375	-0.26228	-0.41088
20	2.5	4.26038246	-0.20964	2.5	2.713473	-0.0875	2.5	1.692146	-0.11178	2.5	0.432015	-0.25674	2.5	-0.31364	-0.40234
21	2.625	4.23417783	-0.18402	2.625	2.702536	-0.06536	2.625	1.678173	-0.09074	2.625	0.399923	-0.24014	2.625	-0.36393	-0.39181
22	2.75	4.21117556	-0.15727	2.75	2.694365	-0.04166	2.75	1.66683	-0.06792	2.75	0.369906	-0.22164	2.75	-0.41291	-0.37955
23	2.875	4.19151635	-0.12977	2.875	2.689158	-0.01675	2.875	1.658341	-0.04366	2.875	0.3422	-0.20157	2.875	-0.46035	-0.36587
24	3	4.17529476	-0.1019	3	2.687063	0.008965	3	1.652883	-0.01834	3	0.317004	-0.18025	3	-0.50609	-0.3511
25	3.125	4.16255729	-0.07405	3.125	2.688184	0.035099	3.125	1.650591	0.007656	3.125	0.294473	-0.15806	3.125	-0.54997	-0.33558
26	3.25	4.15330127	-0.04662	3.25	2.692571	0.061238	3.25	1.651548	0.033926	3.25	0.274715	-0.13535	3.25	-0.59192	-0.31968
27	3.375	4.14747437	-0.02	3.375	2.700226	0.086968	3.375	1.655789	0.060064	3.375	0.257796	-0.11253	3.375	-0.63188	-0.30378
28	3.5	4.14497497	0.005427	3.5	2.711097	0.111882	3.5	1.663297	0.085662	3.5	0.24373	-0.08997	3.5	-0.66985	-0.28825
29	3.625	4.14565332	0.029281	3.625	2.725082	0.135581	3.625	1.674004	0.110317	3.625	0.232484	-0.06806	3.625	-0.70588	-0.2735
30	3.75	4.14931341	0.051218	3.75	2.74203	0.157685	3.75	1.687794	0.133641	3.75	0.223977	-0.04718	3.75	-0.74007	-0.25989
31	3.875	4.15571572	0.070919	3.875	2.76174	0.177835	3.875	1.704499	0.155262	3.875	0.218079	-0.02771	3.875	-0.77256	-0.24781
32	4	4.16458063	0.088095	4	2.78397	0.195703	4	1.723907	0.174836	4	0.214615	-0.00999	4	-0.80353	-0.2376
33	4.125	4.17559255	0.102498	4.125	2.808433	0.210997	4.125	1.745761	0.192047	4.125	0.213367	0.005654	4.125	-0.83323	-0.22962
34	4.25	4.18840475	0.113919	4.25	2.834807	0.223464	4.25	1.769767	0.206618	4.25	0.214073	0.018929	4.25	-0.86194	-0.22416
35	4.375	4.20264467	0.122201	4.375	2.86274	0.232897	4.375	1.795595	0.218313	4.375	0.21644	0.029575	4.375	-0.88996	-0.22151
36	4.5	4.21791976	0.127231	4.5	2.891852	0.239139	4.5	1.822884	0.226942	4.5	0.220136	0.03737	4.5	-0.91764	-0.22193
37	4.625	4.23382368	0.128953	4.625	2.921745	0.242084	4.625	1.851251	0.232364	4.625	0.224808	0.042139	4.625	-0.94539	-0.22563
38	4.75	4.24994279	0.127359	4.75	2.952005	0.241679	4.75	1.880297	0.23449	4.75	0.230075	0.043749	4.75	-0.97359	-0.23278
39	4.875	4.26586272	0.122499	4.875	2.982215	0.237928	4.875	1.909608	0.233287	4.875	0.235544	0.042119	4.875	-1.00269	-0.24352
40	5	4.28117508	0.114471	5	3.011956	0.23089	5	1.938769	0.228775	5	0.240809	0.037216	5	-1.03313	-0.25795
41	5.125	4.29548392	0.103426	5.125	3.040818	0.220677	5.125	1.967366	0.221028	5.125	0.245461	0.029058	5.125	-1.06537	-0.2761
42	5.25	4.30841212	0.089562	5.25	3.068402	0.207453	5.25	1.994994	0.210175	5.25	0.249093	0.017713	5.25	-1.09988	-0.29799
43	5.375	4.31960738	0.073123	5.375	3.094334	0.191432	5.375	2.021266	0.196394	5.375	0.251307	0.003297	5.375	-1.13713	-0.32359
44	5.5	4.32874773	0.054391	5.5	3.118263	0.172874	5.5	2.045816	0.179912	5.5	0.251719	-0.01403	5.5	-1.17758	-0.35282
45	5.625	4.33554657	0.033684	5.625	3.139872	0.152076	5.625	2.068305	0.160999	5.625	0.249966	-0.03405	5.625	-1.22168	-0.38558
46	5.75	4.33975707	0.01135	5.75	3.158881	0.129374	5.75	2.088429	0.139963	5.75	0.245709	-0.05653	5.75	-1.26988	-0.42174
47	5.875	4.34117587	-0.01224	5.875	3.175053	0.10513	5.875	2.105925	0.117147	5.875	0.238642	-0.08119	5.875	-1.3226	-0.46113
48	6	4.33964602	-0.03669	6	3.188194	0.079733	6	2.120568	0.092919	6	0.228493	-0.10771	6	-1.38024	-0.50357
49	6.125	4.33505919	-0.06162	6.125	3.198161	0.053583	6.125	2.132183	0.067669	6.125	0.215029	-0.13576	6.125	-1.44319	-0.54887
50	6.25	4.32735704	-0.0866	6.25	3.204859	0.027095	6.25	2.140642	0.041802	6.25	0.198059	-0.16499	6.25	-1.51179	-0.59681
51	6.375	4.31653183	-0.11125	6.375	3.208246	0.000682	6.375	2.145867	0.01573	6.375	0.177435	-0.19503	6.375	-1.5864	-0.6472
52	6.5	4.30262614	-0.13515	6.5	3.208331	-0.02524	6.5	2.147833	-0.01014	6.5	0.153057	-0.2255	6.5	-1.6673	-0.69983

Figure 1.12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Draw Solutions
Draw Slopes
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max y
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t_0
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Zoom In
Solution
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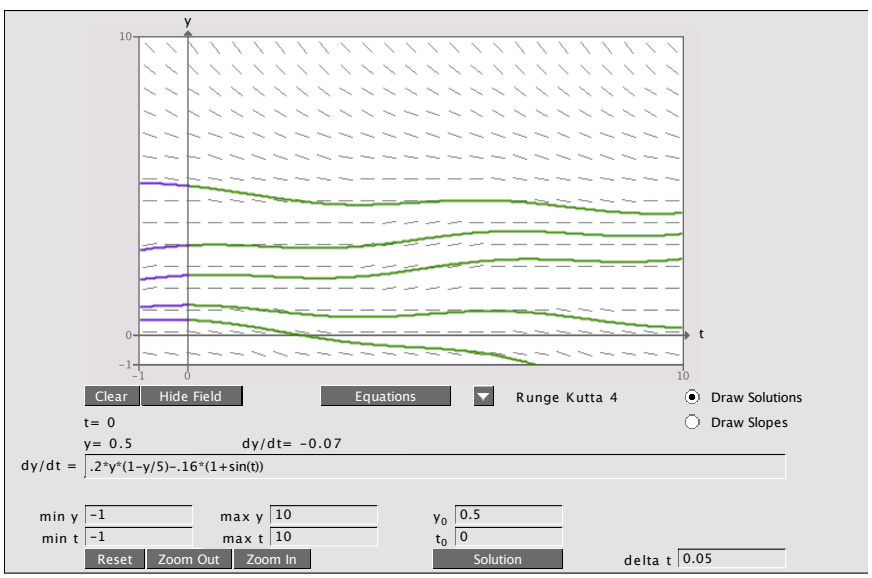
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10
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max t
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t_0
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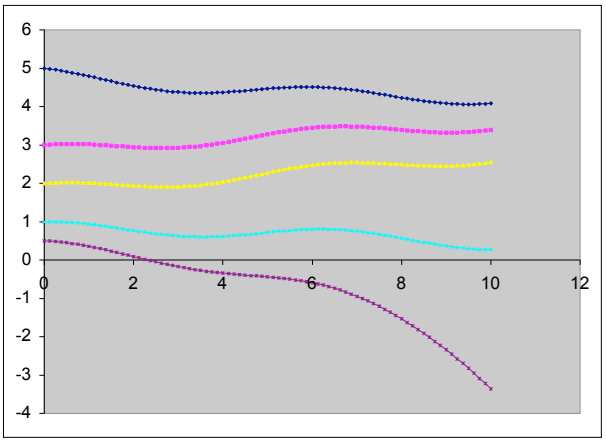
Reset
Zoom Out
Zoom In
Solution
delta t
0.05

Figure 1.13



DELTA T	0.125														
k	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)
0	0	5	-0.16	0	3	0.08	0	2	0.08	0	1	0	0	0.5	-0.07
1	0.125	4.98	-0.17596	0.125	3.01	0.059648	0.125	2.01	0.060448	0.125	1	-0.01995	0.125	0.49125	-0.09135
2	0.25	4.95800451	-0.19126	0.25	3.017456	0.039705	0.25	2.017556	0.041105	0.25	0.997507	-0.03988	0.25	0.479831	-0.11283
3	0.375	4.9340975	-0.2056	0.375	3.022419	0.02048	0.375	2.022694	0.022284	0.375	0.992521	-0.0595	0.375	0.465728	-0.13413
4	0.5	4.90839789	-0.21872	0.5	3.024979	0.002268	0.5	2.02548	0.004285	0.5	0.985083	-0.07851	0.5	0.448961	-0.15498
5	0.625	4.88105748	-0.23039	0.625	3.025263	-0.01465	0.625	2.026015	-0.0126	0.625	0.97527	-0.09661	0.625	0.429589	-0.17508
6	0.75	4.85225836	-0.24039	0.75	3.023431	-0.03002	0.75	2.02444	-0.02811	0.75	0.963194	-0.11353	0.75	0.407704	-0.19417
7	0.875	4.82220999	-0.24851	0.875	3.019678	-0.04361	0.875	2.020926	-0.04199	0.875	0.949002	-0.12903	0.875	0.383432	-0.212
8	1	4.79114582	-0.25461	1	3.014227	-0.05521	1	2.015678	-0.05402	1	0.932873	-0.14287	1	0.356932	-0.22834
9	1.125	4.75931966	-0.25854	1.125	3.007326	-0.06466	1.125	2.008926	-0.06401	1.125	0.915014	-0.15485	1.125	0.328389	-0.243
10	1.25	4.72700168	-0.26022	1.25	2.999243	-0.07181	1.25	2.000925	-0.0718	1.25	0.895658	-0.16479	1.25	0.298014	-0.25579
11	1.375	4.6944743	-0.25957	1.375	2.990267	-0.07656	1.375	1.99195	-0.07727	1.375	0.875059	-0.17256	1.375	0.266041	-0.26657
12	1.5	4.66202785	-0.25657	1.5	2.980698	-0.07884	1.5	1.982291	-0.08032	1.5	0.853489	-0.17804	1.5	0.23272	-0.27522
13	1.625	4.62995613	-0.25123	1.625	2.970843	-0.07863	1.625	1.972251	-0.08091	1.625	0.831234	-0.18116	1.625	0.198317	-0.28167
14	1.75	4.59855194	-0.24359	1.75	2.961013	-0.07594	1.75	1.962138	-0.07901	1.75	0.808589	-0.18187	1.75	0.163108	-0.28588
15	1.875	4.56810262	-0.23374	1.875	2.951521	-0.07081	1.875	1.952262	-0.07465	1.875	0.785855	-0.18019	1.875	0.127373	-0.28783
16	2	4.53888566	-0.22177	2	2.94267	-0.06333	2	1.94293	-0.0679	2	0.763332	-0.17613	2	0.091394	-0.28754
17	2.125	4.51116444	-0.20784	2.125	2.934754	-0.05361	2.125	1.934442	-0.05885	2.125	0.741316	-0.16977	2.125	0.055452	-0.28508
18	2.25	4.48518413	-0.19213	2.25	2.928053	-0.04182	2.25	1.927087	-0.04762	2.25	0.720095	-0.16121	2.25	0.019816	-0.28054
19	2.375	4.46116789	-0.17484	2.375	2.922825	-0.02814	2.375	1.921134	-0.03439	2.375	0.699943	-0.1506	2.375	-0.01525	-0.27405
20	2.5	4.43931329	-0.15619	2.5	2.919308	-0.01279	2.5	1.916835	-0.01936	2.5	0.681118	-0.13809	2.5	-0.04951	-0.26576
21	2.625	4.41978917	-0.13645	2.625	2.917709	0.003994	2.625	1.914415	-0.00274	2.625	0.663857	-0.12388	2.625	-0.08273	-0.25585
22	2.75	4.40273281	-0.11588	2.75	2.918208	0.021938	2.75	1.914072	0.015202	2.75	0.648372	-0.10821	2.75	-0.11471	-0.24453
23	2.875	4.38824763	-0.09477	2.875	2.920951	0.040761	2.875	1.915972	0.034205	2.875	0.634846	-0.0913	2.875	-0.14527	-0.23205
24	3	4.37640131	-0.07341	3	2.926046	0.06016	3	1.920248	0.053976	3	0.623433	-0.07344	3	-0.17428	-0.21865
25	3.125	4.3672245	-0.05212	3.125	2.933566	0.079826	3.125	1.926995	0.074212	3.125	0.614253	-0.0549	3.125	-0.20161	-0.2046
26	3.25	4.36071003	-0.03118	3.25	2.943544	0.099442	3.25	1.936271	0.0946	3.25	0.607391	-0.03597	3.25	-0.22719	-0.19019
27	3.375	4.35681272	-0.0109	3.375	2.955974	0.118691	3.375	1.948096	0.114823	3.375	0.602895	-0.01695	3.375	-0.25096	-0.1757
28	3.5	4.35544983	0.008418	3.5	2.97081	0.137259	3.5	1.962449	0.134567	3.5	0.600776	0.001843	3.5	-0.27292	-0.16144
29	3.625	4.35650202	0.026504	3.625	2.987968	0.154843	3.625	1.97927	0.153521	3.625	0.601006	0.020121	3.625	-0.2931	-0.14769
30	3.75	4.359815	0.043093	3.75	3.007323	0.171155	3.75	1.99846	0.171388	3.75	0.603522	0.037585	3.75	-0.31157	-0.13475
31	3.875	4.36520167	0.057946	3.875	3.028718	0.185923	3.875	2.019884	0.187884	3.875	0.60822	0.053951	3.875	-0.32841	-0.12289
32	4	4.37244488	0.070846	4	3.051958	0.198902	4	2.043369	0.202748	4	0.614964	0.068954	4	-0.34377	-0.11239
33	4.125	4.38130068	0.081611	4.125	3.076821	0.209874	4.125	2.068713	0.215742	4.125	0.623583	0.082345	4.125	-0.35782	-0.1035
34	4.25	4.39150203	0.090087	4.25	3.103055	0.218651	4.25	2.095681	0.226659	4.25	0.633876	0.093902	4.25	-0.37076	-0.09645
35	4.375	4.40276292	0.096159	4.375	3.130386	0.225084	4.375	2.124013	0.235325	4.375	0.645614	0.10343	4.375	-0.38281	-0.09145
36	4.5	4.41478283	0.099749	4.5	3.158522	0.229059	4.5	2.153429	0.2416	4.5	0.658542	0.110766	4.5	-0.39424	-0.08866
37	4.625	4.42725146	0.100818	4.625	3.187154	0.230502	4.625	2.183629	0.245386	4.625	0.672388	0.115783	4.625	-0.40533	-0.08825
38	4.75	4.43985365	0.099366	4.75	3.215967	0.229382	4.75	2.214302	0.246622	4.75	0.686861	0.118388	4.75	-0.41636	-0.09032
39	4.875	4.45227435	0.095434	4.875	3.24464	0.22571	4.875	2.24513	0.245291	4.875	0.701659	0.118528	4.875	-0.42765	-0.09496
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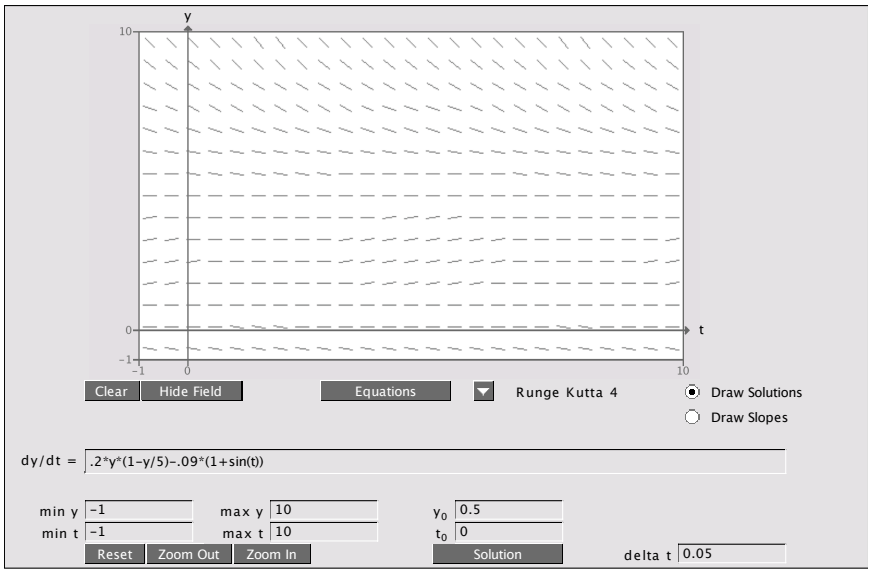
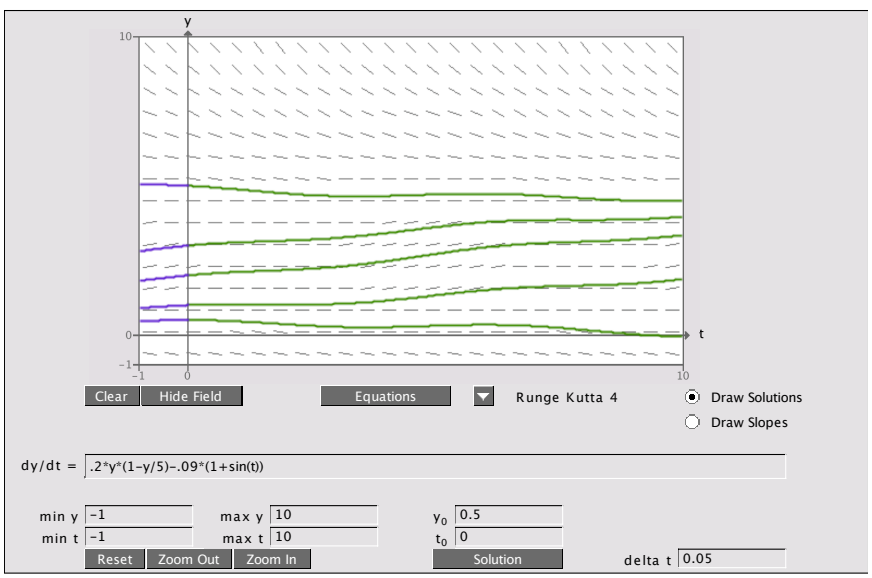
41	5.125	4.47534164	0.080493	5.125	3.300295	0.210953	5.125	2.305968	0.235066	5.125	0.730999	0.111398	5.125	-0.45229	-0.11207
42	5.25	4.4854033	0.069756	5.25	3.326665	0.200095	5.25	2.335352	0.226345	5.25	0.744924	0.104218	5.25	-0.4663	-0.12453
43	5.375	4.49412286	0.057081	5.375	3.351676	0.187128	5.375	2.363645	0.215398	5.375	0.757951	0.094753	5.375	-0.48187	-0.13952
44	5.5	4.501258	0.042685	5.5	3.375067	0.172257	5.5	2.390569	0.202407	5.5	0.769795	0.083142	5.5	-0.49931	-0.15695
45	5.625	4.50659364	0.026812	5.625	3.3966	0.155714	5.625	2.41587	0.187586	5.625	0.780188	0.069559	5.625	-0.51893	-0.17669
46	5.75	4.5099452	0.009729	5.75	3.416064	0.137758	5.75	2.439319	0.171177	5.75	0.788883	0.054208	5.75	-0.54101	-0.19859
47	5.875	4.51116138	-0.00828	5.875	3.433283	0.11867	5.875	2.460716	0.153449	5.875	0.795659	0.03732	5.875	-0.56584	-0.22246
48	6	4.51012642	-0.02692	6	3.448117	0.098749	6	2.479897	0.13469	6	0.800324	0.01915	6	-0.59364	-0.24812
49	6.125	4.50676169	-0.04588	6.125	3.460461	0.078305	6.125	2.496733	0.115204	6.125	0.802718	-2.6E-05	6.125	-0.62466	-0.27534
50	6.25	4.50102675	-0.06486	6.25	3.470249	0.057653	6.25	2.511134	0.095304	6.25	0.802714	-0.01992	6.25	-0.65907	-0.30388
51	6.375	4.4929198	-0.08354	6.375	3.477456	0.037113	6.375	2.523047	0.075309	6.375	0.800224	-0.04024	6.375	-0.69706	-0.33352
52	6.5	4.48247743	-0.10163	6.5	3.482095	0.017	6.5	2.53246	0.055539	6.5	0.795194	-0.06067	6.5	-0.73875	-0.364

Figure 1.14


y
10
0
-1
-1
0
10
t
Clear
Hide Field
Equations
Runge Kutta 4
Draw Solutions
Draw Slopes
dy/dt =
.2*y*(1-y/5)-.09*(1+sin(t))
min y
-1
max y
10
y_0
0.5
min t
-1
max t
10
t_0
0
Reset
Zoom Out
Zoom In
Solution
delta t
0.05


y
10
0
-1
-1
0
10
t
Clear
Hide Field
Equations
Runge Kutta 4
Draw Solutions
Draw Slopes
dy/dt =
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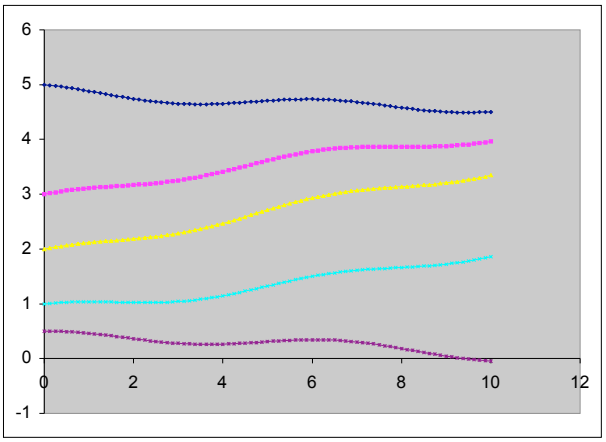
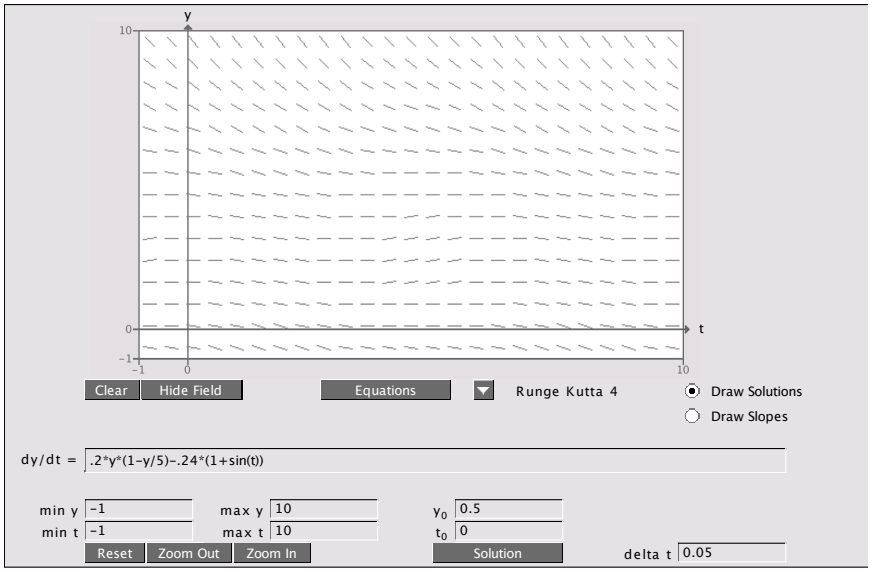
min y
-1
max y
10
y_0
0.5
min t
-1
max t
10
t_0
0
Reset
Zoom Out
Zoom In
Solution
delta t
0.05

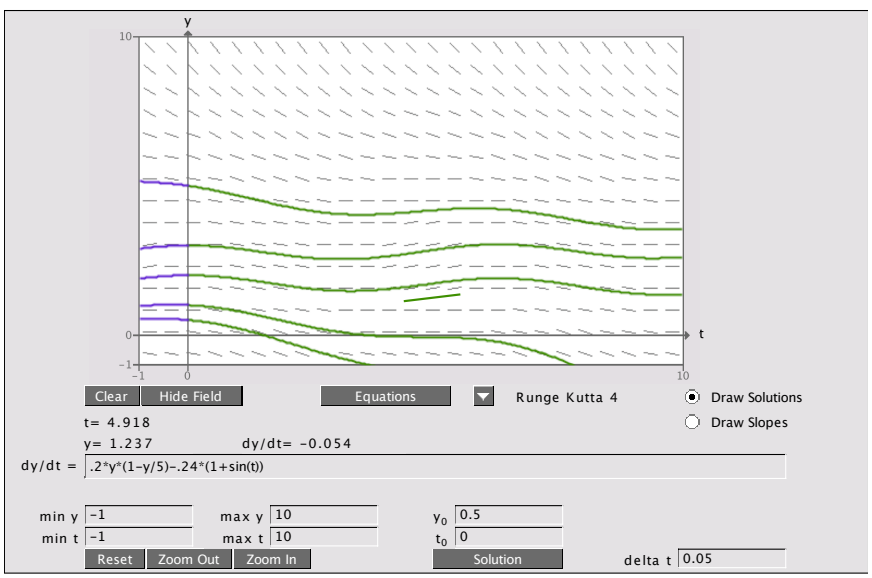
Figure 1.15



DELTA T	0.125														
k	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)
0	0	5	-0.09	0	3	0.15	0	2	0.15	0	1	0.07	0	0.5	0
1	0.125	4.98875	-0.09898	0.125	3.01875	0.138015	0.125	2.01875	0.139515	0.125	1.00875	0.059826	0.125	0.5	-0.01122
2	0.25	4.97637803	-0.10756	0.25	3.036002	0.126242	0.25	2.036189	0.129129	0.25	1.016228	0.049671	0.25	0.498597	-0.02249
3	0.375	4.96293249	-0.11561	0.375	3.051782	0.114857	0.375	2.052331	0.119019	0.375	1.022437	0.039708	0.375	0.495786	-0.03364
4	0.5	4.94848174	-0.12295	0.5	3.066139	0.104031	0.5	2.067208	0.109359	0.5	1.027401	0.03011	0.5	0.491581	-0.0445
5	0.625	4.93311289	-0.12946	0.625	3.079143	0.093925	0.625	2.080878	0.100315	0.625	1.031164	0.021042	0.625	0.486019	-0.0549
6	0.75	4.91693036	-0.13501	0.75	3.090884	0.084687	0.75	2.093417	0.09204	0.75	1.033795	0.012662	0.75	0.479156	-0.0647
7	0.875	4.90005416	-0.13949	0.875	3.10147	0.07645	0.875	2.104922	0.084678	0.875	1.035377	0.005116	0.875	0.471068	-0.07374
8	1	4.88261799	-0.14281	1	3.111026	0.069334	1	2.115507	0.078354	1	1.036017	-0.00146	1	0.461851	-0.08189
9	1.125	4.8647671	-0.14489	1.125	3.119693	0.063435	1.125	2.125301	0.07318	1.125	1.035834	-0.00696	1.125	0.451614	-0.08904
10	1.25	4.84665597	-0.14568	1.25	3.127622	0.058835	1.25	2.134449	0.069246	1.25	1.034965	-0.01126	1.25	0.440484	-0.09507
11	1.375	4.82844593	-0.14515	1.375	3.134976	0.055592	1.375	2.143104	0.066625	1.375	1.033557	-0.0143	1.375	0.4286	-0.09991
12	1.5	4.81030258	-0.14327	1.5	3.141925	0.053743	1.5	2.151433	0.065365	1.5	1.03177	-0.016	1.5	0.416111	-0.10348
13	1.625	4.79239327	-0.14007	1.625	3.148643	0.053303	1.625	2.159603	0.065497	1.625	1.029769	-0.01633	1.625	0.403177	-0.10573
14	1.75	4.77488446	-0.13556	1.75	3.155306	0.054264	1.75	2.16779	0.067027	1.75	1.027728	-0.01526	1.75	0.38996	-0.10665
15	1.875	4.75793912	-0.1298	1.875	3.162089	0.056598	1.875	2.176169	0.069938	1.875	1.02582	-0.0128	1.875	0.376629	-0.10622
16	2	4.74171421	-0.12285	2	3.169164	0.060252	2	2.184911	0.074192	2	1.024221	-0.00895	2	0.363352	-0.10445
17	2.125	4.7263582	-0.1148	2.125	3.176695	0.065155	2.125	2.194185	0.07973	2.125	1.023101	-0.00378	2.125	0.350296	-0.10138
18	2.25	4.71200875	-0.10575	2.25	3.18484	0.071213	2.25	2.204151	0.086472	2.25	1.022629	0.002668	2.25	0.337623	-0.09706
19	2.375	4.69879051	-0.09582	2.375	3.193741	0.078317	2.375	2.21496	0.094318	2.375	1.022963	0.010303	2.375	0.325491	-0.09157
20	2.5	4.68681316	-0.08515	2.5	3.203531	0.086339	2.5	2.22675	0.103151	2.5	1.024251	0.019024	2.5	0.314044	-0.085
21	2.625	4.67616958	-0.07388	2.625	3.214323	0.095137	2.625	2.239644	0.112836	2.625	1.026629	0.028714	2.625	0.303419	-0.07745
22	2.75	4.66693441	-0.06217	2.75	3.226215	0.104555	2.75	2.253748	0.123225	2.75	1.030218	0.03924	2.75	0.293738	-0.06905
23	2.875	4.6591627	-0.05019	2.875	3.239285	0.114428	2.875	2.269151	0.134158	2.875	1.035123	0.050455	2.875	0.285106	-0.05994
24	3	4.65288902	-0.0381	3	3.253588	0.124583	3	2.285921	0.145466	3	1.04143	0.062202	3	0.277614	-0.05026
25	3.125	4.64812676	-0.02607	3.125	3.269161	0.134842	3.125	2.304105	0.156972	3.125	1.049205	0.074314	3.125	0.271331	-0.04017
26	3.25	4.64486786	-0.01428	3.25	3.286017	0.145025	3.25	2.323726	0.168495	3.25	1.058494	0.08662	3.25	0.26631	-0.02984
27	3.375	4.64308276	-0.0029	3.375	3.304145	0.154951	3.375	2.344788	0.179853	3.375	1.069322	0.098943	3.375	0.26258	-0.01943
28	3.5	4.6427208	0.00792	3.5	3.323513	0.164444	3.5	2.367269	0.190866	3.5	1.08169	0.111106	3.5	0.260152	-0.00911
29	3.625	4.64371085	0.018012	3.625	3.344069	0.173334	3.625	2.391128	0.201358	3.625	1.095578	0.122936	3.625	0.259014	0.000951
30	3.75	4.64596235	0.027234	3.75	3.365736	0.181461	3.75	2.416297	0.21116	3.75	1.110945	0.134262	3.75	0.259133	0.010581
31	3.875	4.64936665	0.035455	3.875	3.388418	0.188675	3.875	2.442692	0.220115	3.875	1.127728	0.144921	3.875	0.260455	0.019624
32	4	4.65379857	0.042558	4	3.412003	0.194842	4	2.470207	0.228077	4	1.145843	0.154763	4	0.262908	0.027929
33	4.125	4.65911835	0.048443	4.125	3.436358	0.199845	4.125	2.498716	0.234915	4.125	1.165188	0.163646	4.125	0.266399	0.035356
34	4.25	4.66517379	0.05303	4.25	3.461338	0.203582	4.25	2.528081	0.240518	4.25	1.185644	0.171448	4.25	0.270819	0.041779
35	4.375	4.67180253	0.056257	4.375	3.486786	0.205976	4.375	2.558145	0.244791	4.375	1.207075	0.17806	4.375	0.276041	0.047086
36	4.5	4.67883465	0.058085	4.5	3.512533	0.206969	4.5	2.588744	0.247663	4.5	1.229332	0.183394	4.5	0.281927	0.051184
37	4.625	4.68609526	0.058496	4.625	3.538404	0.206525	4.625	2.619702	0.249083	4.625	1.252257	0.187382	4.625	0.288325	0.053996
38	4.75	4.69340727	0.057495	4.75	3.56422	0.204634	4.75	2.650838	0.249026	4.75	1.275679	0.189978	4.75	0.295074	0.055468
39	4.875	4.70059413	0.055108	4.875	3.589799	0.201306	4.875	2.681966	0.247488	4.875	1.299427	0.191158	4.875	0.302008	0.055566
40	5	4.70748265	0.051384	5	3.614962	0.196578	5	2.712902	0.24449	5	1.323321	0.19092	5	0.308954	0.054276
41	5.125	4.71390565	0.046392	5.125	3.639535	0.190505	5.125	2.743463	0.240076	5.125	1.347186	0.189288	5.125	0.315738	0.051607
42	5.25	4.71970462	0.040221	5.25	3.663348	0.183169	5.25	2.773473	0.234313	5.25	1.370847	0.186305	5.25	0.322189	0.04759
43	5.375	4.72473219	0.032978	5.375	3.686244	0.174668	5.375	2.802762	0.227288	5.375	1.394135	0.182038	5.375	0.328138	0.042276
44	5.5	4.7288544	0.024787	5.5	3.708077	0.165121	5.5	2.831173	0.219112	5.5	1.41689	0.176574	5.5	0.333422	0.035736
45	5.625	4.73195277	0.015787	5.625	3.728717	0.154662	5.625	2.858562	0.209909	5.625	1.438962	0.170019	5.625	0.337889	0.028063
46	5.75	4.73392613	0.006128	5.75	3.74805	0.14344	5.75	2.8848	0.199822	5.75	1.460214	0.162499	5.75	0.341397	0.019362
47	5.875	4.73469214	-0.00403	5.875	3.76598	0.131617	5.875	2.909778	0.189008	5.875	1.480527	0.154152	5.875	0.343817	0.00976
48	6	4.73418852	-0.01452	6	3.782432	0.119362	6	2.933404	0.177634	6	1.499796	0.145131	6	0.345037	-0.00061
49	6.125	4.73237395	-0.02516	6.125	3.797353	0.106852	6.125	2.955608	0.165874	6.125	1.517937	0.135599	6.125	0.344962	-0.01159
50	6.25	4.72922866	-0.03579	6.25	3.810709	0.094268	6.25	2.976343	0.15391	6.25	1.534887	0.125728	6.25	0.343513	-0.02303
51	6.375	4.72475462	-0.04623	6.375	3.822493	0.081789	6.375	2.995581	0.141924	6.375	1.550603	0.115694	6.375	0.340634	-0.03477
52	6.5	4.71897549	-0.05631	6.5	3.832716	0.069594	6.5	3.013322	0.130099	6.5	1.565065	0.105675	6.5	0.336288	-0.04663

Figure 1.16


y
10
0
-1
-1
0
10
t
Clear
Hide Field
Equations
Runge Kutta 4
Draw Solutions
Draw Slopes
dy/dt =
.2*y*(1-y/5)-.24*(1+sin(t))
min y
-1
max y
10
y_0
0.5
min t
-1
max t
10
t_0
0
Reset
Zoom Out
Zoom In
Solution
delta t
0.05


y
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0
-1
-1
0
10
t
Clear
Hide Field
Equations
Runge Kutta 4
Draw Solutions
Draw Slopes
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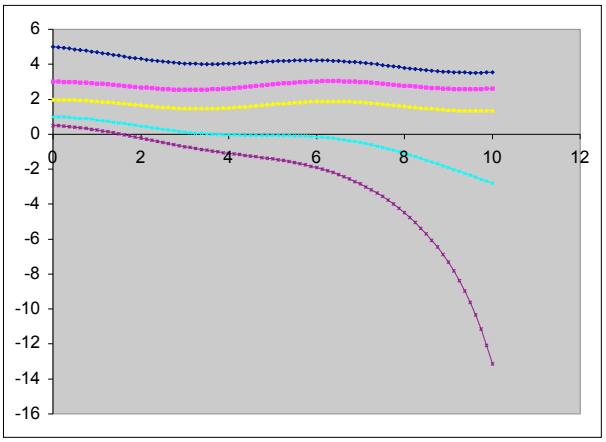
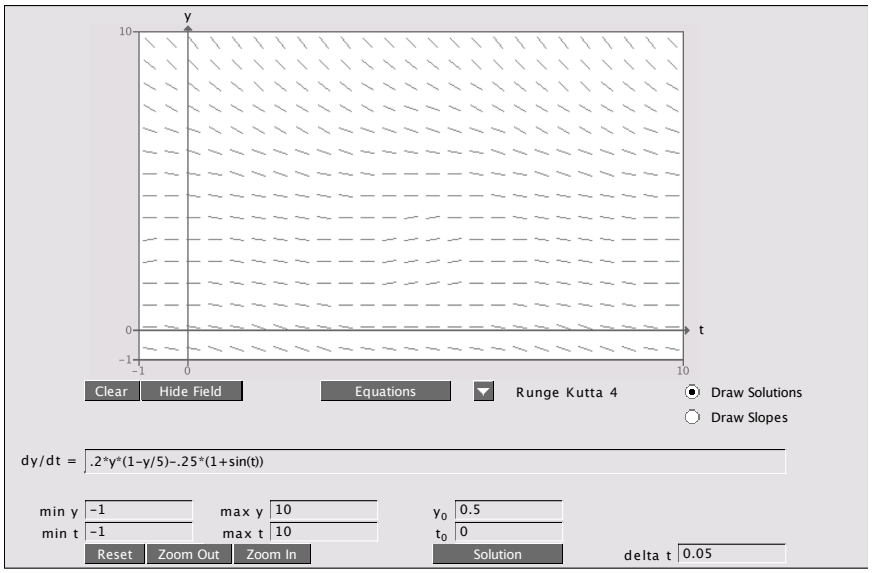
y= 1.237
dy/dt= -0.054
dy/dt =
.2*y*(1-y/5)-.24*(1+sin(t))
min y
-1
max y
10
y_0
0.5
min t
-1
max t
10
t_0
0
Reset
Zoom Out
Zoom In
Solution
delta t
0.05

Figure 1.17



DELTA T	0.125														
k	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)
0	0	5	-0.24	0	3	0	0	2	0	0	1	-0.08	0	0.5	-0.15
1	0.125	4.97	-0.26396	0.125	3	-0.02992	0.125	2	-0.02992	0.125	0.99	-0.11113	0.125	0.48125	-0.18294
2	0.25	4.93700526	-0.28694	0.25	2.99626	-0.05923	0.25	1.99626	-0.05953	0.25	0.976109	-0.14227	0.25	0.458383	-0.2161
3	0.375	4.90113817	-0.30852	0.375	2.988856	-0.08746	0.375	1.988819	-0.08836	0.375	0.958326	-0.17298	0.375	0.43137	-0.24907
4	0.5	4.86257267	-0.32833	0.5	2.977923	-0.1142	0.5	1.977774	-0.11597	0.5	0.936704	-0.20282	0.5	0.400236	-0.28142
5	0.625	4.82153115	-0.346	0.625	2.963648	-0.13902	0.625	1.963278	-0.14195	0.625	0.911352	-0.23138	0.625	0.365058	-0.31274
6	0.75	4.7782807	-0.36122	0.75	2.946271	-0.16156	0.75	1.945535	-0.16589	0.75	0.88243	-0.25825	0.75	0.325965	-0.34265
7	0.875	4.73312872	-0.37368	0.875	2.926076	-0.18147	0.875	1.924798	-0.18744	0.875	0.850148	-0.28309	0.875	0.283134	-0.37079
8	1	4.6864181	-0.38317	1	2.903392	-0.19846	1	1.901368	-0.20629	1	0.814762	-0.30555	1	0.236785	-0.39684
9	1.125	4.63852185	-0.38948	1.125	2.878584	-0.21228	1.125	1.875582	-0.22214	1.125	0.776567	-0.32535	1.125	0.18718	-0.42051
10	1.25	4.58983744	-0.39245	1.25	2.852049	-0.22271	1.25	1.847814	-0.23477	1.25	0.735898	-0.34224	1.25	0.134616	-0.44156
11	1.375	4.5407808	-0.39201	1.375	2.82421	-0.22962	1.375	1.818468	-0.24399	1.375	0.693118	-0.35601	1.375	0.079422	-0.45978
12	1.5	4.49178008	-0.38809	1.5	2.795508	-0.23289	1.5	1.787969	-0.24968	1.5	0.648617	-0.3665	1.5	0.021949	-0.47503
13	1.625	4.44326929	-0.3807	1.625	2.766396	-0.23249	1.625	1.756759	-0.25174	1.625	0.602805	-0.37362	1.625	-0.03743	-0.48719
14	1.75	4.39568187	-0.3699	1.75	2.737335	-0.22841	1.75	1.725291	-0.25016	1.75	0.556102	-0.37731	1.75	-0.09833	-0.49621
15	1.875	4.34944424	-0.3558	1.875	2.708784	-0.22072	1.875	1.69402	-0.24496	1.875	0.508939	-0.37755	1.875	-0.16035	-0.50208
16	2	4.30496945	-0.33855	2	2.681194	-0.20954	2	1.6634	-0.23623	2	0.461744	-0.37441	2	-0.22311	-0.50485
17	2.125	4.26265095	-0.31835	2.125	2.655001	-0.19504	2.125	1.633871	-0.22408	2.125	0.414943	-0.36798	2.125	-0.28622	-0.5046
18	2.25	4.22285667	-0.29547	2.25	2.630621	-0.17742	2.25	1.605861	-0.20872	2.25	0.368946	-0.35839	2.25	-0.3493	-0.50148
19	2.375	4.1859233	-0.27018	2.375	2.608443	-0.15695	2.375	1.579771	-0.19036	2.375	0.324147	-0.34586	2.375	-0.41198	-0.49567
20	2.5	4.15215106	-0.24282	2.5	2.588824	-0.13395	2.5	1.555977	-0.16928	2.5	0.280915	-0.33061	2.5	-0.47394	-0.48741
21	2.625	4.12179888	-0.21375	2.625	2.57208	-0.10875	2.625	1.534817	-0.1458	2.625	0.239589	-0.31292	2.625	-0.53486	-0.47696
22	2.75	4.09508011	-0.18337	2.75	2.558487	-0.08174	2.75	1.516591	-0.12028	2.75	0.200474	-0.29311	2.75	-0.59448	-0.46463
23	2.875	4.07215888	-0.15209	2.875	2.54827	-0.05332	2.875	1.501556	-0.0931	2.875	0.163835	-0.27153	2.875	-0.65256	-0.45077
24	3	4.05314708	-0.12036	3	2.541605	-0.02394	3	1.489918	-0.06468	3	0.129893	-0.24857	3	-0.70891	-0.43575
25	3.125	4.03810215	-0.08861	3.125	2.538613	0.005958	3.125	1.481833	-0.03545	3.125	0.098823	-0.22461	3.125	-0.76338	-0.41997
26	3.25	4.0270256	-0.05731	3.25	2.539357	0.035905	3.25	1.477402	-0.00586	3.25	0.070747	-0.20008	3.25	-0.81587	-0.40383
27	3.375	4.01986242	-0.02689	3.375	2.543845	0.065434	3.375	1.476669	0.023622	3.375	0.045736	-0.17543	3.375	-0.86635	-0.38778
28	3.5	4.01650133	0.002197	3.5	2.552025	0.09408	3.5	1.479622	0.052541	3.5	0.023808	-0.15107	3.5	-0.91483	-0.37225
29	3.625	4.01677594	0.029527	3.625	2.563785	0.121389	3.625	1.48619	0.080439	3.625	0.004924	-0.12746	3.625	-0.96136	-0.35769
30	3.75	4.02046686	0.054702	3.75	2.578958	0.146925	3.75	1.496245	0.106874	3.75	-0.01101	-0.10503	3.75	-1.00607	-0.34453
31	3.875	4.0273046	0.077351	3.875	2.597324	0.170278	3.875	1.509604	0.131422	3.875	-0.02414	-0.08419	3.875	-1.04913	-0.3332
32	4	4.03697345	0.097141	4	2.618609	0.19107	4	1.526032	0.153688	4	-0.03466	-0.06535	4	-1.09078	-0.32412
33	4.125	4.04911609	0.113783	4.125	2.642492	0.208962	4.125	1.545243	0.173311	4.125	-0.04283	-0.04887	4.125	-1.1313	-0.31768
34	4.25	4.063339	0.127036	4.25	2.668613	0.22366	4.25	1.566906	0.189971	4.25	-0.04894	-0.03509	4.25	-1.17101	-0.31425
35	4.375	4.07921854	0.136712	4.375	2.69657	0.234924	4.375	1.590653	0.203393	4.375	-0.05333	-0.02431	4.375	-1.21029	-0.31418
36	4.5	4.09630755	0.142679	4.5	2.725936	0.242565	4.5	1.616077	0.213354	4.5	-0.05636	-0.01679	4.5	-1.24956	-0.31776
37	4.625	4.11414246	0.144866	4.625	2.756256	0.246457	4.625	1.642746	0.219689	4.625	-0.05846	-0.01275	4.625	-1.28928	-0.32526
38	4.75	4.1322507	0.143261	4.75	2.787064	0.246534	4.75	1.670207	0.222288	4.75	-0.06006	-0.01233	4.75	-1.32994	-0.33691
39	4.875	4.15015828	0.137913	4.875	2.81788	0.242792	4.875	1.697993	0.221105	4.875	-0.0616	-0.01564	4.875	-1.37206	-0.35288
40	5	4.1673974	0.128933	5	2.848229	0.235291	5	1.725632	0.216156	5	-0.06355	-0.02273	5	-1.41616	-0.37331
41	5.125	4.18351406	0.11649	5.125	2.877641	0.224154	5.125	1.752651	0.207517	5.125	-0.06639	-0.0336	5.125	-1.46283	-0.3983
42	5.25	4.19807527	0.100806	5.25	2.90566	0.209562	5.25	1.778591	0.195327	5.25	-0.07059	-0.04817	5.25	-1.51262	-0.4279
43	5.375	4.21067601	0.082157	5.375	2.931855	0.191753	5.375	1.803007	0.179781	5.375	-0.07661	-0.06634	5.375	-1.5661	-0.46211
44	5.5	4.2209456	0.060864	5.5	2.955824	0.171019	5.5	1.825479	0.161131	5.5	-0.08491	-0.08794	5.5	-1.62387	-0.50092
45	5.625	4.22855355	0.037288	5.625	2.977202	0.147695	5.625	1.845621	0.139675	5.625	-0.0959	-0.11274	5.625	-1.68648	-0.54426
46	5.75	4.23321453	0.011826	5.75	2.995664	0.12216	5.75	1.86308	0.11576	5.75	-0.10999	-0.1405	5.75	-1.75452	-0.59205
47	5.875	4.23469274	-0.0151	5.875	3.010933	0.094825	5.875	1.87755	0.089769	5.875	-0.12755	-0.17089	5.875	-1.82852	-0.64418
48	6	4.23280527	-0.04304	6	3.022787	0.066127	6	1.888771	0.062116	6	-0.14892	-0.20361	6	-1.90904	-0.70053
49	6.125	4.22742467	-0.07155	6.125	3.031052	0.036526	6.125	1.896536	0.03324	6.125	-0.17437	-0.23828	6.125	-1.99661	-0.76097
50	6.25	4.21848048	-0.10016	6.25	3.035618	0.006488	6.25	1.90069	0.003596	6.25	-0.20415	-0.27453	6.25	-2.09173	-0.8254
51	6.375	4.20595998	-0.12842	6.375	3.036429	-0.02351	6.375	1.90114	-0.02635	6.375	-0.23847	-0.31197	6.375	-2.19491	-0.89369
52	6.5	4.18990791	-0.15586	6.5	3.03349	-0.05301	6.5	1.897846	-0.05613	6.5	-0.27747	-0.3502	6.5	-2.30662	-0.96577

Figure 1.18



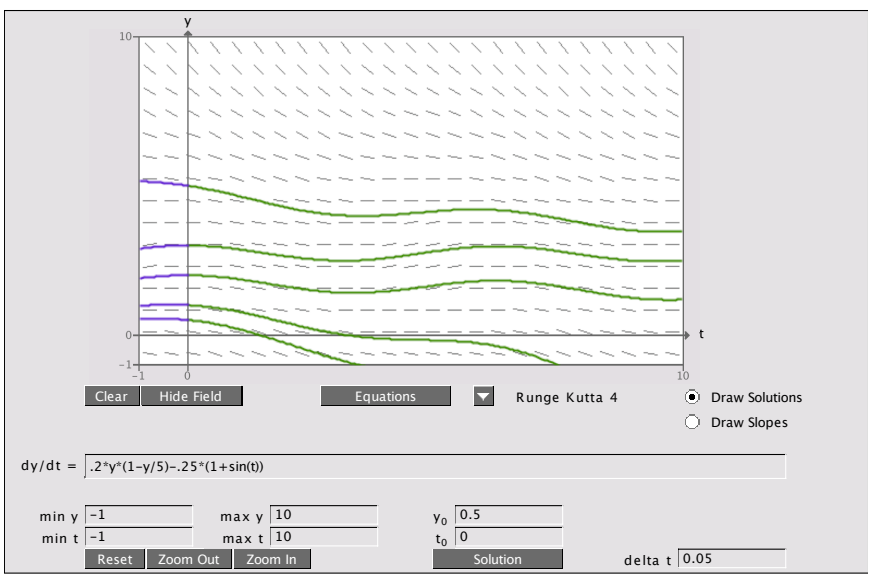
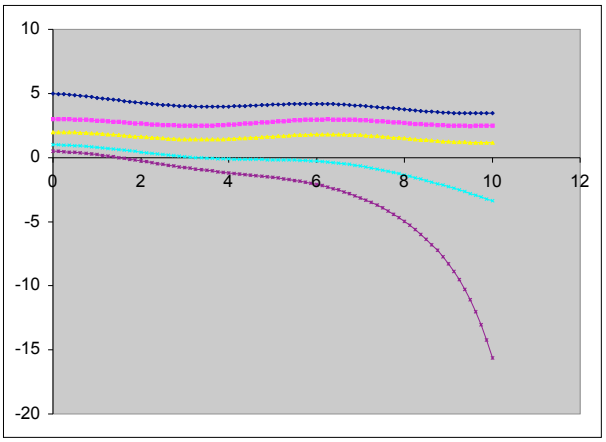


Figure 1.19



DELTA T	0.125														
k	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)	t_k	y_k	f(t_k,y_k)
0	0	5	-0.25	0	3	-0.01	0	2	-0.01	0	1	-0.09	0	0.5	-0.16
1	0.125	4.96875	-0.27496	0.125	2.99875	-0.04112	0.125	1.99875	-0.04122	0.125	0.98875	-0.12252	0.125	0.48	-0.19438
2	0.25	4.93438028	-0.2989	0.25	2.99361	-0.0716	0.25	1.993598	-0.07211	0.25	0.973435	-0.15507	0.25	0.455702	-0.22902
3	0.375	4.89701787	-0.3214	0.375	2.984661	-0.10096	0.375	1.984584	-0.10219	0.375	0.954051	-0.18717	0.375	0.427075	-0.26345
4	0.5	4.85684338	-0.34204	0.5	2.97204	-0.12877	0.5	1.97181	-0.13102	0.5	0.930655	-0.21837	0.5	0.394144	-0.29724
5	0.625	4.81408778	-0.36047	0.625	2.955944	-0.15459	0.625	1.955433	-0.15814	0.625	0.903359	-0.24824	0.625	0.356988	-0.32997
6	0.75	4.76902848	-0.37635	0.75	2.93662	-0.17804	0.75	1.935666	-0.18315	0.75	0.872328	-0.27638	0.75	0.315742	-0.36125
7	0.875	4.72198482	-0.38937	0.875	2.914366	-0.19875	0.875	1.912772	-0.20568	0.875	0.837781	-0.3024	0.875	0.270586	-0.3907
8	1	4.673313	-0.3993	1	2.889522	-0.21644	1	1.887062	-0.2254	1	0.79998	-0.32597	1	0.221748	-0.41798
9	1.125	4.62340058	-0.40592	1.125	2.862467	-0.23082	1.125	1.858888	-0.24201	1.125	0.759234	-0.34678	1.125	0.1695	-0.44282
10	1.25	4.57266057	-0.40908	1.25	2.833614	-0.2417	1.25	1.828637	-0.25528	1.25	0.715887	-0.36457	1.25	0.114148	-0.46494
11	1.375	4.52152519	-0.40869	1.375	2.803402	-0.24891	1.375	1.796727	-0.26501	1.375	0.670316	-0.37913	1.375	0.056031	-0.48414
12	1.5	4.47043946	-0.40468	1.5	2.772289	-0.25234	1.5	1.763602	-0.27107	1.5	0.622924	-0.39031	1.5	-0.00449	-0.50027
13	1.625	4.41985459	-0.39707	1.625	2.740746	-0.25195	1.625	1.729718	-0.27337	1.625	0.574135	-0.39799	1.625	-0.06702	-0.51322
14	1.75	4.37022127	-0.38591	1.75	2.709252	-0.24775	1.75	1.695548	-0.27188	1.75	0.524386	-0.40212	1.75	-0.13117	-0.52292
15	1.875	4.32198308	-0.37131	1.875	2.678284	-0.23979	1.875	1.661562	-0.26664	1.875	0.474121	-0.40269	1.875	-0.19654	-0.52937
16	2	4.27556978	-0.35343	2	2.64831	-0.2282	2	1.628232	-0.25772	2	0.423785	-0.39975	2	-0.26271	-0.53263
17	2.125	4.231391	-0.33249	2.125	2.619784	-0.21315	2.125	1.596017	-0.24527	2.125	0.373816	-0.39341	2.125	-0.32929	-0.53277
18	2.25	4.18982993	-0.30874	2.25	2.59314	-0.19487	2.25	1.565358	-0.22946	2.25	0.324641	-0.38381	2.25	-0.39588	-0.52996
19	2.375	4.15123752	-0.28248	2.375	2.568782	-0.17361	2.375	1.536676	-0.21054	2.375	0.276665	-0.37115	2.375	-0.46213	-0.52439
20	2.5	4.11592693	-0.25407	2.5	2.547081	-0.14971	2.5	1.510358	-0.18879	2.5	0.230271	-0.35568	2.5	-0.52768	-0.51629
21	2.625	4.08416858	-0.22386	2.625	2.528367	-0.12351	2.625	1.486759	-0.16455	2.625	0.185811	-0.3377	2.625	-0.59222	-0.50595
22	2.75	4.05618562	-0.19228	2.75	2.512928	-0.09542	2.75	1.466191	-0.13817	2.75	0.143598	-0.31752	2.75	-0.65546	-0.49369
23	2.875	4.03215015	-0.15976	2.875	2.501	-0.06586	2.875	1.44892	-0.11005	2.875	0.103908	-0.29551	2.875	-0.71717	-0.47987
24	3	4.01218004	-0.12675	3	2.492768	-0.03528	3	1.435164	-0.08064	3	0.066969	-0.27207	3	-0.77715	-0.46487
25	3.125	3.9963366	-0.09371	3.125	2.488358	-0.00415	3.125	1.425084	-0.05037	3.125	0.032961	-0.2476	3.125	-0.83526	-0.44911
26	3.25	3.98462298	-0.06112	3.25	2.487838	0.027043	3.25	1.418788	-0.01971	3.25	0.002011	-0.22255	3.25	-0.8914	-0.43302
27	3.375	3.97698355	-0.02944	3.375	2.491219	0.05782	3.375	1.416324	0.010849	3.375	-0.02581	-0.19736	3.375	-0.94553	-0.41704
28	3.5	3.97330408	0.000871	3.5	2.498446	0.087696	3.5	1.417681	0.040839	3.5	-0.05048	-0.1725	3.5	-0.99766	-0.40165
29	3.625	3.97341293	0.029362	3.625	2.509408	0.116196	3.625	1.422785	0.069784	3.625	-0.07204	-0.14842	3.625	-1.04787	-0.38729
30	3.75	3.97708317	0.055619	3.75	2.523933	0.142867	3.75	1.431508	0.097223	3.75	-0.09059	-0.12556	3.75	-1.09628	-0.37444
31	3.875	3.98403559	0.079257	3.875	2.541791	0.167281	3.875	1.443661	0.122717	3.875	-0.10629	-0.10436	3.875	-1.14308	-0.36353
32	4	3.99394268	0.099926	4	2.562701	0.189043	4	1.459001	0.145853	4	-0.11933	-0.08524	4	-1.18852	-0.35501
33	4.125	4.00643343	0.117324	4.125	2.586332	0.2078	4.125	1.477233	0.166256	4.125	-0.12999	-0.06858	4.125	-1.2329	-0.34928
34	4.25	4.02109893	0.131198	4.25	2.612307	0.223243	4.25	1.498015	0.183588	4.25	-0.13856	-0.05473	4.25	-1.27656	-0.34675
35	4.375	4.03749864	0.141349	4.375	2.640212	0.235119	4.375	1.520963	0.197565	4.375	-0.1454	-0.04402	4.375	-1.3199	-0.34776
36	4.5	4.05516732	0.147641	4.5	2.669602	0.243232	4.5	1.545659	0.207952	4.5	-0.1509	-0.03671	4.5	-1.36337	-0.35264
37	4.625	4.07362241	0.149995	4.625	2.700006	0.247446	4.625	1.571653	0.214573	4.625	-0.15549	-0.03302	4.625	-1.40745	-0.36168
38	4.75	4.09237172	0.148397	4.75	2.730937	0.24769	4.75	1.598474	0.217313	4.75	-0.15962	-0.03312	4.75	-1.45266	-0.37512
39	4.875	4.11092138	0.142899	4.875	2.761898	0.243958	4.875	1.625639	0.216122	4.875	-0.16376	-0.03712	4.875	-1.49955	-0.39316
40	5	4.12878379	0.133614	5	2.792393	0.236311	5	1.652654	0.211011	5	-0.1684	-0.04508	5	-1.5487	-0.41595
41	5.125	4.14548549	0.120714	5.125	2.821932	0.224874	5.125	1.67903	0.20206	5.125	-0.17403	-0.057	5.125	-1.60069	-0.44361
42	5.25	4.16057478	0.104433	5.25	2.850041	0.209832	5.25	1.704288	0.189407	5.25	-0.18116	-0.07281	5.25	-1.65614	-0.47621
43	5.375	4.17362894	0.085056	5.375	2.87627	0.191434	5.375	1.727964	0.173256	5.375	-0.19026	-0.0924	5.375	-1.71567	-0.51378
44	5.5	4.18426092	0.062916	5.5	2.900199	0.169979	5.5	1.749621	0.153862	5.5	-0.20181	-0.11561	5.5	-1.77989	-0.55631
45	5.625	4.19212538	0.038389	5.625	2.921446	0.145816	5.625	1.768853	0.131538	5.625	-0.21626	-0.1422	5.625	-1.84943	-0.60378
46	5.75	4.19692401	0.011888	5.75	2.939673	0.119337	5.75	1.785295	0.106638	5.75	-0.23404	-0.17193	5.75	-1.9249	-0.65612
47	5.875	4.19840998	-0.01615	5.875	2.954591	0.09097	5.875	1.798625	0.079559	5.875	-0.25553	-0.20448	5.875	-2.00692	-0.71326
48	6	4.19639151	-0.04526	6	2.965962	0.061169	6	1.80857	0.050731	6	-0.28109	-0.23952	6	-2.09607	-0.7751
49	6.125	4.19073452	-0.07496	6.125	2.973608	0.030409	6.125	1.814911	0.020608	6.125	-0.31103	-0.27669	6.125	-2.19296	-0.84157
50	6.25	4.1813643	-0.10478	6.25	2.977409	-0.00082	6.25	1.817487	-0.01034	6.25	-0.34562	-0.31561	6.25	-2.29816	-0.9126
51	6.375	4.16826623	-0.13425	6.375	2.977306	-0.03203	6.375	1.816195	-0.04163	6.375	-0.38507	-0.35587	6.375	-2.41223	-0.98812
52	6.5	4.15148548	-0.16288	6.5	2.973302	-0.06274	6.5	1.810992	-0.07277	6.5	-0.42955	-0.39707	6.5	-2.53575	-1.06813