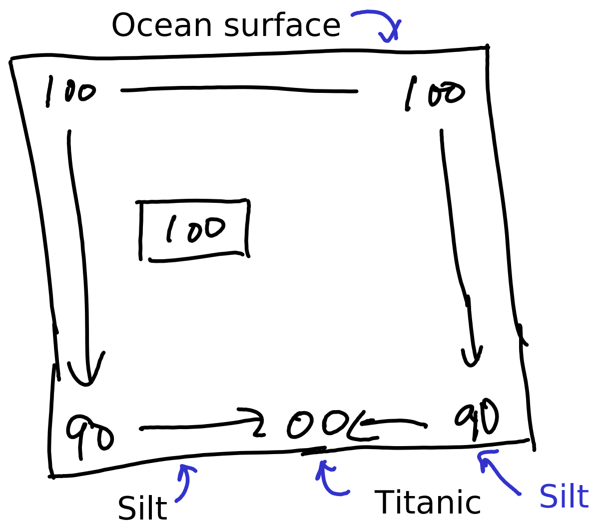
Lecture 31 Hmwk review

Titanic problem

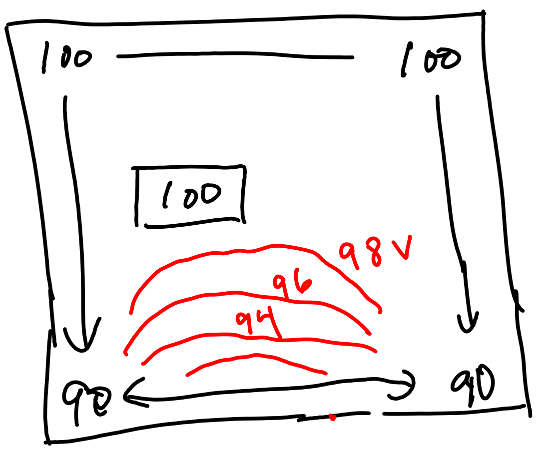


Set the boundary values to determine current flow out of that boundary.

100 V at the probe and at the water surface leads to no voltage between these points and therefore

$$\Delta V = 0 = IR \Rightarrow I = 0$$

Between the probe and silt on the bottom of the ocean set the voltage at the bottom boundary to be say 90.

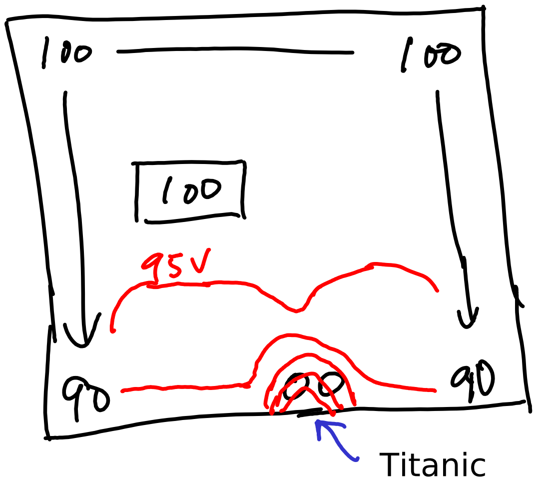


The Titanic is not in this region.

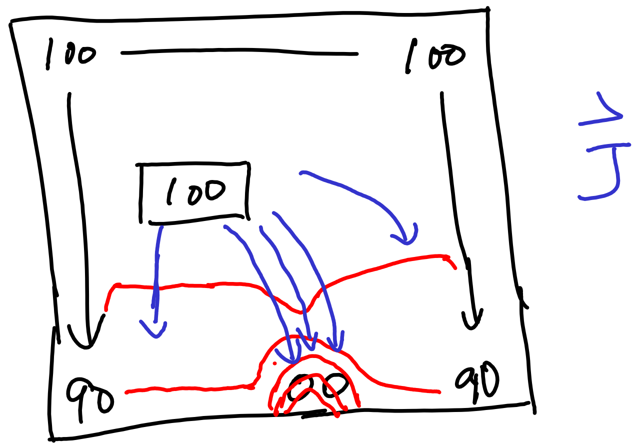
$$\Delta V = 10 = IR$$ The current is small.

E is perpendicular to constant voltage contours.

$$\vec{E} = -\vec{\nabla} V \qquad \vec{J} = \sigma \vec{E}$$



This is analogous to the contour plot for a hill. The slope goes downward gently to the 90 V region. The Titanic is down a steep slope.



There is a discontinuity in the voltage between the Titanic at 0 V and the silt immediately next to it at 90 V. This is an unphysical boundary condition since

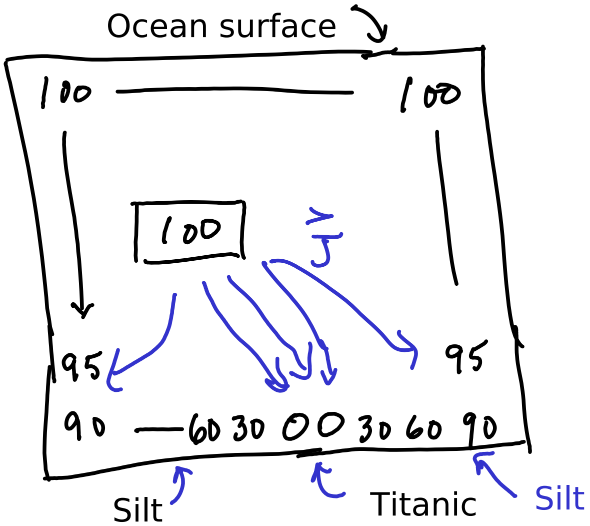
$$\vec{E} = -\hat{x}\frac{\partial}{\partial x}V - \hat{y}\frac{\partial}{\partial y}V - \hat{z}\frac{\partial}{\partial z}V$$

looks like $\frac{\Delta V}{\Delta x} = \frac{90-0}{\Delta x} \to \infty$ as $\Delta x \to 0$

An infinite E leads to an infinite J in Ohms law: $J = \sigma E$

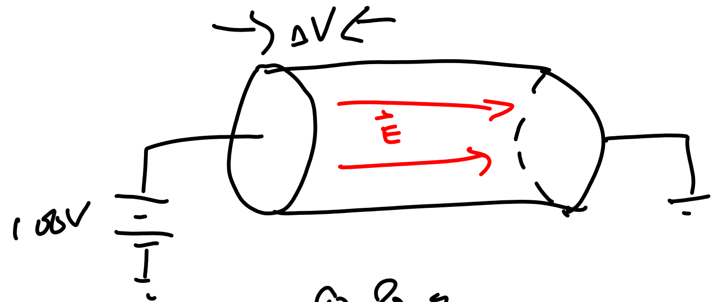
Numerical soln may give an answer but you have to be careful that what you ask it to model is correctly linked to a physical situation.

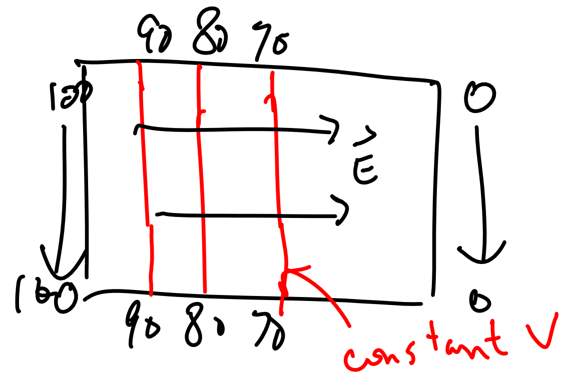
Boundary conditions that match a real situation would like like



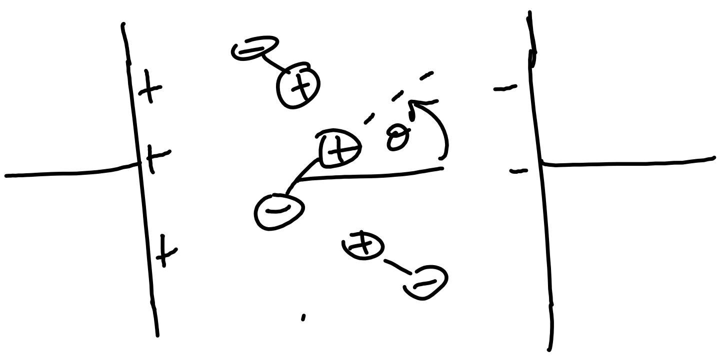
Here is the boundary condition on a copper wire

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$





Dipole moment per vol for a gas of water molecules in a cap



$$\vec{P} = \int n(\theta)\, p_0 \cos\theta \, d\Omega = N \, p^2 \, \frac{\vec{E}}{3kT}$$

N = # molecules / vol

p = dipole mom / molecule = qs

Questions:

-modify: What happens if the electric field is oscillatory?

-incongruous: How can this be right since when the temperature goes to zero P becomes infinite?

-informational: How well does this model agree with data?

-analogy: What is the analogous situation for magnetic materials?

-modifying: How do I calculate P for a liquid or solid?

-modifying: How do I calculate P for induced dipoles?

-causal/creative: The effect of each dipole on the others is not part of this model. How do we model this?

-causal/creative: What relativitic effects need to be taken into account (in a star)?

↓

more

$$\vec{P} = N p^2 \frac{\vec{E}}{3kT}$$

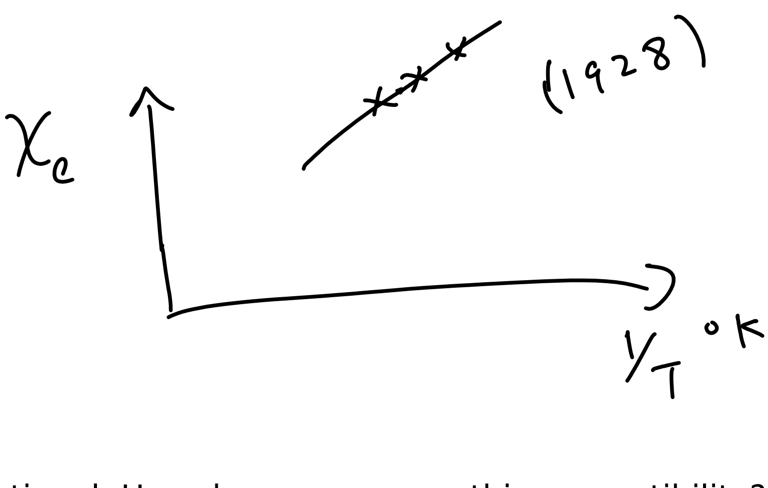
N: $\frac{\text{\# molecules}}{\text{vol}}$; p: $\frac{\text{dipole mom}}{\text{molecule}} = qs$

For many materials the model P proportional E works well.

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

χ_e ← electric susceptibility

$$\chi_e = \frac{P}{\epsilon_0 E} = \frac{Np^2}{2kT\epsilon_0}$$

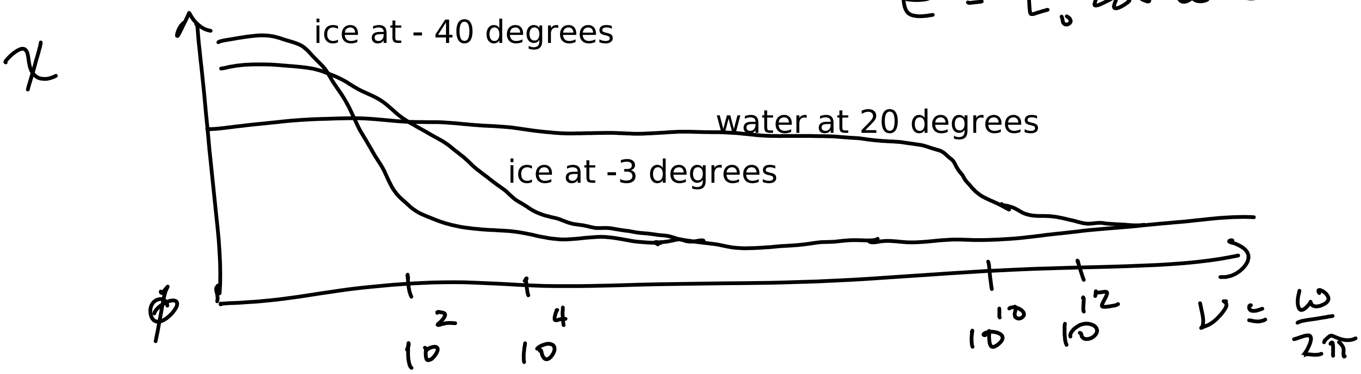


For steam at fixed N

informational: How do you measure this susceptibility?

The value of the capacitance is affected by the material.

$E = E_0 \cos \omega t$



Questions:

incongruous: How can this be right if the susceptibility does NOT go to zero?

incongruous: How can this be right since a water molecule has the same dipole moment in ice or water?

causal/creative: How can the model take into account the moment of inertia of the water molecule?

I expect there to be some interpretation issues in the homework and for you to contact me to clarify those issues.

I mentioned last time that to get a feeling of how small atoms are just put a small amount end to end and see how long such a string could be.

Questions:

Congruous: How do you calculate the length of the string?

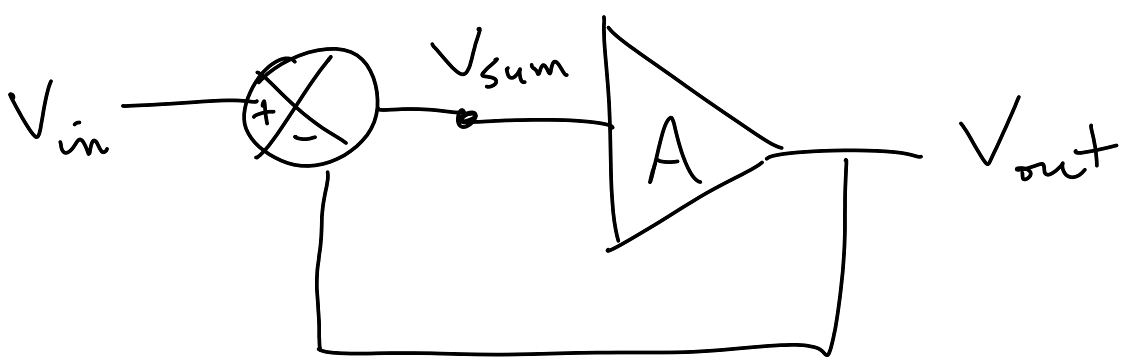
$$10^{23} \text{ atoms}, \quad 10^{-10} \frac{\text{m}}{\text{atom}} \Rightarrow 10^{13} \text{ m}$$

Distance from the sun to Pluto is 6×10^{12} m

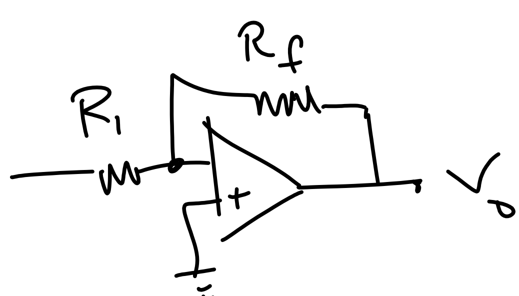
Time for light to travel from the sun to pluto is 5.5 hours.

Feedback and retardation effects in circuits

Block diagram of an amplifier with feedback

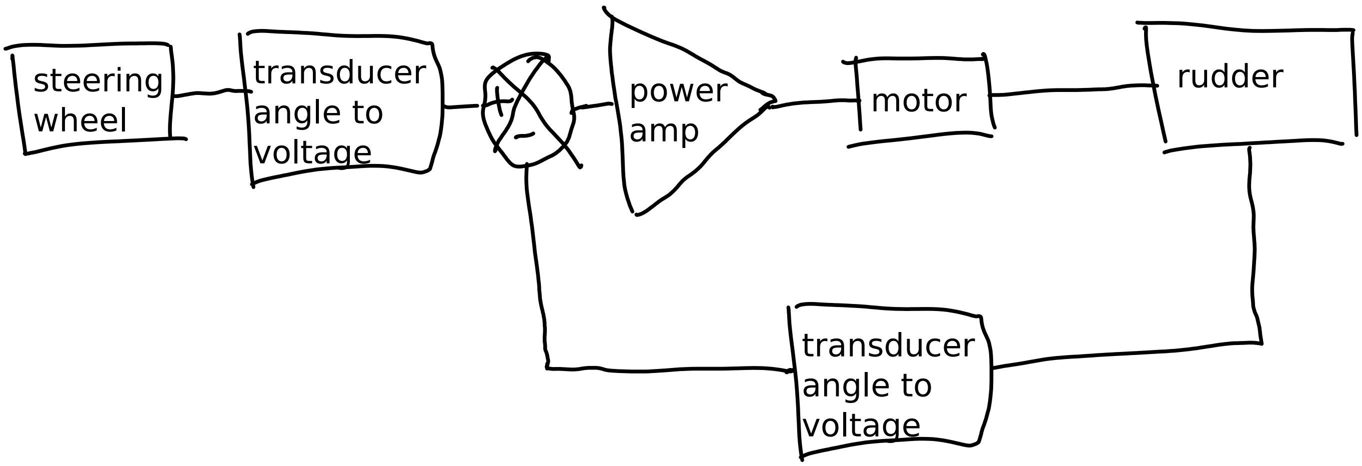


Actual circuit



I want to illustrate how retardation effects feedback.

Block diagram of power steering (airplane rudder etc)



$V_{wheel} - V_{rudder}$ goes into amp

In steady state this voltage difference is zero.

Now the temperature increases making the amp reducing the power out of the amp and there for the angle of the rudder.

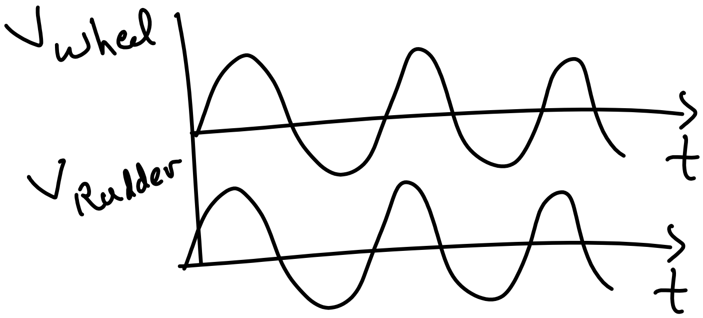
$$V_{wheel} - V_{rudder} > 0$$

This positive error causes the amp to put out more power thereby moving the rudder to a position where this voltage difference is zero.

The same thing happens if you turn the wheel to a larger angle: the difference in voltage causes the amp to put out more power and move the rudder until this voltage difference is zero.

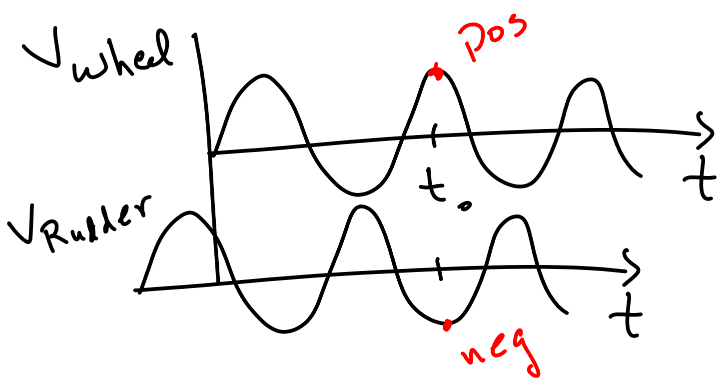
Now turn the vehicle left and right and left and right etc.

There will be a time delay due to the inertia of the rudder, delay in circuits etc



This retarded time effect is called lag and lead time in control theory.

The retardation time can be large enough for this



V_{wheel} V_{Rudder}
$$\text{pos} - \text{neg} = \text{pos feedback}$$

This positive feedback generates oscillation of the system.

$$V_{wheel} - V_{Rudder}\Big|_{t_0}$$

is positive even though the wheel angle and rudder angle are the same. This positive voltage then drives the rudder away when it should do nothing.

Thse feedback effects occur in you body both chemically and mechanically (e.g. when you try to grab something. Feedback from you eyes helps get you fingers on that object.