Phys 361 Homework 11

1a) You guys want to see a neat trick? Calculate the vector potential at a point some distance r away from an infinite straight wire with some current I.

b) You probably got a divergent integral. That's bad. But not unrecoverable. Try this: When you go to integrate along the length of the wire, instead of letting your bounds be $-\infty$ to $+\infty$, double your integrand and integrate from 0 to R. Write the vector potential you get.

c) Ok, so now let R get big. Not quite infinite, but let $R \gg r$. Write the resulting potential and take the curl of it. Or take the curl of your potential and then let R get big; whichever floats your boat. Are we in a good place now?

d) Justify what we just did. Why was it okay? Or, if you can't convince yourself it was okay, argue for why it was inappropriate.

2) (based on Pollack & Stump 8.28)

a) Let's suppose we have a very long cylindrical conducting wire of radius R_0 laying right along the z-axis. The wire carries a uniformly distributed current I_0 in the +z direction. Find the magnetic field inside and outside the wire.

b) Now let's drill a cylindrical hole down the length of the wire, parallel to the z axis, but offset from it. The center of the hole is at $x = a$ and radius of the hole is b. The hole sits entirely inside of the original wire. The wire still carries the <u>same total current</u> as before. Find the magnetic field in the hole. Hint: The field in the hole should work out to be uniform. Another hint: A region of space with no current is the same as a region of space with current in the $+z$ direction superimposed on the same region with current going in the $-z$ direction.

Note: The technique you get to use here is one of very general applicability, since lots of things look like fairly symmetric objects with bits removed (or added, for that matter). Superposition is your friend.

3) (based on Pollack and Stump 8.22)

In class we (painfully) derived the vector potential associated with a spinning spherical shell of charge. Use that vector potential to find the magnetic field everywhere inside and outside the shell. Then sketch the field (by hand or in Mathematica) and describe what you get. If you sketch the field by hand, you might find it useful to get oriented by comparing your equation to the equation for the field made by a dipole.

4) The previous problem got us used to working with a spinning sphere of charge as a source of magnetic fields. You've probably been doing spin in quantum mechanics lately, or will be pretty soon. Sometimes we try to classically conceptualize spin as the result of a tiny ball of charge literally spinning. The mass of the ball leads to the angular momentum (spin) of the elementary particle, and the charge of the ball leads to the magnetic moment of the elementary particle. This is, as you may already know, totally inappropriate. Electrons aren't actually little rotating balls. Spin is intrinsic angular momentum that's just there and if you don't like it, too bad. Nature doesn't care what you think. Ditto with the intrinsic magnetic moment of a particle. Nevertheless, we'll try out a classical model of spin.

a) Given a spherical shell of radius a and surface mass density σ_m rotating at some angular speed ω, figure out its spin angular momentum. Let the shell be centered on the origin, since classical angular momentum is origin-dependent.

b) Given the same shell with surface charge density σ_q, figure out its magnetic dipole moment.

c) Now let's model an electron as a spinning shell of charge (something we've done before). Your answer to b) should depend on the radius of the shell. Experimentally we've been able to determine the radius of an electron to be at most 10^{-22} m (many people believe it to be exactly zero, which is an odd possibility to ponder).[1] We've also managed to measure the electron's magnetic dipole moment to be $-928\text{x}10^{-26}$ J/T. If we plug this information into b), how fast would the electron have to be spinning if it is in fact a classical shell of charge? Note that things only get worse if the electron turns out to have a radius smaller than that.

d) Ok, so the very small size of the electron is kind of an issue in this model. But, we can make that go away by taking the ratio of the electron's spin magnetic moment to its spin angular momentum. Then quite a lot of stuff cancels. What do we get for that ratio, symbolically, using our shell model? Troll around on the internet and find out what the actual ratio turns out to be. How did our model do this time?

5) (based on Pollack and Stump 8.24)

We haven't talked much about the magnetic field of the Earth yet. It turns out to be more complicated than we might at first think: it doesn't only point north. The Earth's magnetic field comes from its rotating liquid core, which is deep below the surface and makes a roughly dipole-ish field. Depending on where you are on the surface of the Earth, the field might point mostly up or down rather than mostly north or south.

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[1] If you root around on the interwebs, you might find a number called the classical electron radius, which is on the order of 10^{-15}. This number has nothing to do with the measured physical radius of the electron – it's a calculated value that represents how big the electron would have to be for its mass energy (mc^2) to be the same as the energy contained in its electric field.

It's a pretty good approximation to say that the magnetic field comes from a point dipole at the center of the earth, with the dipole moment given by:

$$\vec{m}_E = m_E(\sin\theta_0 \cos\varphi_0\, \hat{\imath} + \sin\theta_0 \sin\varphi_0\, \hat{\jmath} + \cos\theta_0\, \hat{k})$$

Where $m_E = 7.79 \times 10^{22}\ \mathrm{A \cdot m^2}$, and θ_0 and φ_0 are constants equal to 169 degrees and 109 degrees respectively. The *z* axis is the Earth's rotation axis and the *x* axis passes through the prime meridian. Positive φ is to the east.

a) Using the above dipole moment and the equation for the magnetic field of a dipole, find the magnetic field at some arbitrary point on the surface of the Earth with colatitude θ and longitude φ. Give me the field components north (in the $-\hat{\theta}$ direction), to the east ($\hat{\varphi}$), and vertically ($\hat{r}$).

b) Find the field at the longitude and latitude corresponding to your home town. Report the numbers in Gauss. A Gauss is a small unit of field ($1\ \mathrm{G} = 10^{-4}\ \mathrm{T}$), and it's not too unusual to see small fields (like the Earth's) reported in Gauss.