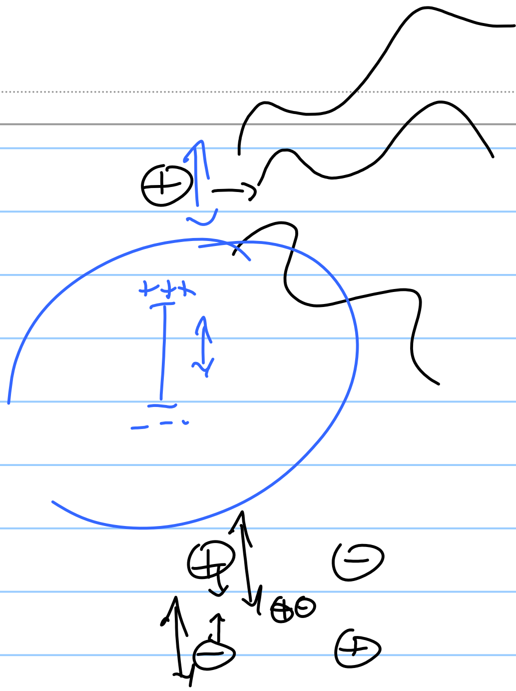
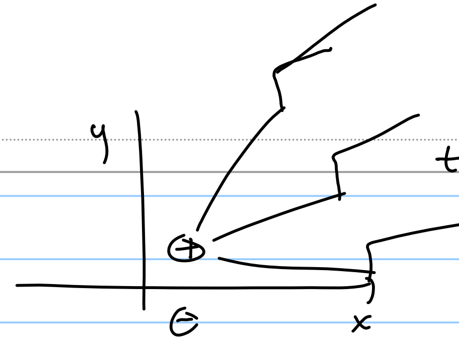
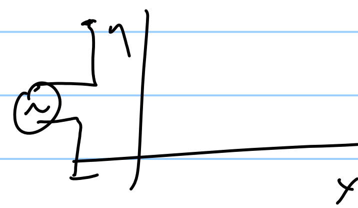
Note Title 7/5/2006

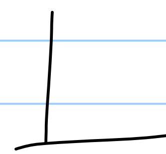




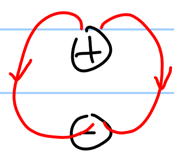
$t=0$



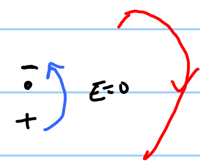
$t=1$



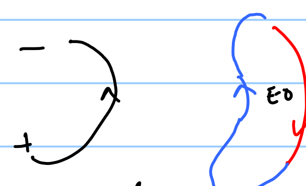
A



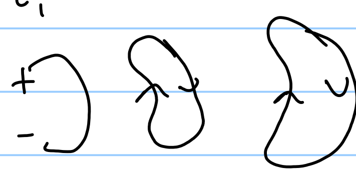
$t=0$



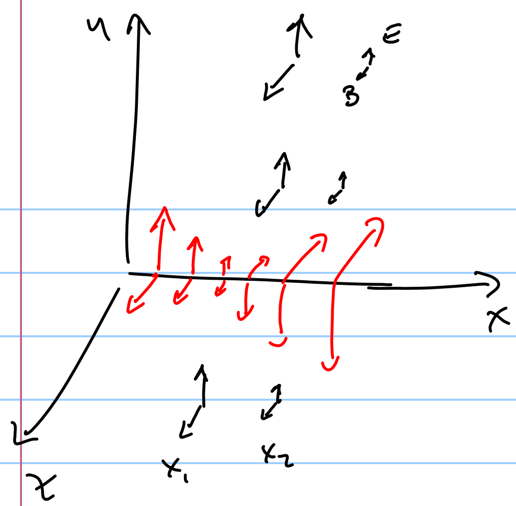
t_1



t_2



t_3



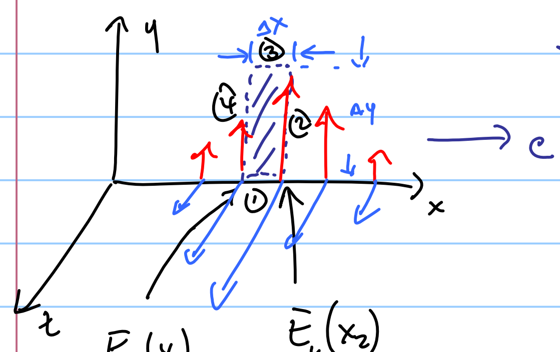
plane EM wave going along x-axis: every point in a plane $\parallel$ to y-z plane E & B are the same

$$\vec{E} = \vec{E}_0 \cos(kx-\omega t)\,\hat{y}$$

$$\vec{B} = \vec{B}_0 \cos(kx-\omega t)\,\hat{z}$$

$$= \vec{B}_0 e^{i(kx-\omega t)}\,\hat{z}$$

How does the EM field distribution satisfy ME's?



fix location of small area

$$\mathcal{E}mf = -\frac{d\Phi_B}{dt}$$

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$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} = -\frac{\partial}{\partial t}\int_0^{\Delta y}\!\!\int_0^{\Delta x} B_z\, da$$

$$= -\frac{\partial}{\partial t} B_z(\Delta x\Delta y)$$

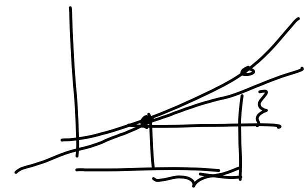
$$\underbrace{\int_{1} \vec{E}\cdot d\vec{l}}_{0} + \int_{2} + \underbrace{\int_{3}}_{0} + \int_{4} = \underbrace{\int_0^{\Delta y} E_y(x_2)\,dy}_{2} + \underbrace{\int_{\Delta y}^{0} E_y(x_1)\,dy}_{4}$$

small Δy

''

$\vec{E} \perp d\vec{l}$

$$E_y(x_2) = E_y(x_1) + \frac{\partial E_y}{\partial x}\Big|_{\text{hold } y,\, z,\, t \text{ fixed}} \Delta x + \cdots$$

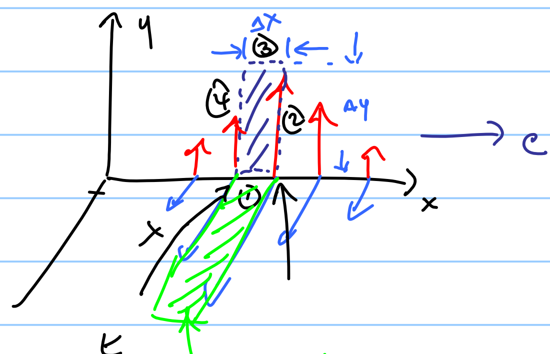


$$\int_{(2)} = \left(E_y(x_1) + \frac{\partial E_y}{\partial x}\Delta x\right)\Delta y \qquad \int_{(4)} = E_y(x_1)\,\Delta y$$

$$\int_{(2)} - \int_{(4)} = \frac{\partial E_y}{\partial x}\cancel{\Delta x\,\Delta y} = -\frac{\partial B_z}{\partial t}\cancel{\Delta x\,\Delta y}$$

I $\quad \boxed{\dfrac{\partial E_y}{\partial x} = -\dfrac{\partial B_z}{\partial t}}$

II $\quad \boxed{\dfrac{\partial}{\partial} = \dfrac{\partial}{\partial}}$



flux of E

Ampere's law

$$\nabla\times\vec{B} = \mu_0 \cancel{J} + \mu_0 J_D$$

I + II

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$ wave eqn

$E_y = c B_z$ for ME to be satisfied

Lorentz force: $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$

$$\vec{B} = \vec{\nabla}\times\vec{A} \qquad \vec{E} = -\frac{\partial A}{\partial t} - \vec{\nabla} V$$

$$\vec{\nabla}(\vec{A}\cdot\vec{B}) = \vec{A}\times\overbrace{(\vec{\nabla}\times\vec{B})}^{\frac{\partial}{\partial x} v = 0} + \vec{B}\times(\vec{\nabla}\times\vec{A}) + \underbrace{(\vec{A}\cdot\vec{\nabla})B}_{\frac{\partial}{\partial x} v = 0}$$
$$+ (\vec{B}\cdot\vec{\nabla})\vec{A}$$

$$\vec{F} = q\left[-\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t} + \vec{v}\times(\vec{\nabla}\times\vec{A})\right]$$

$$\vec{r} = x(t)\hat{x} + y(t)\hat{y} + z(t)\hat{z} \qquad \vec{r} = v_{0x}\hat{x} - \frac{1}{2}gt^2\hat{y}$$

$$\frac{d\vec{r}}{dt} = \vec{v}\,(\text{time})$$

convective derivative

$d\vec{A}/dt$

$$\vec F = \frac{d\vec p}{dt} = -q\left[\frac{\partial \vec A}{\partial t} + (\vec v\cdot\vec\nabla)\vec A + \vec\nabla(V - \vec v\cdot\vec A)\right]$$ Lorentz force in terms potential

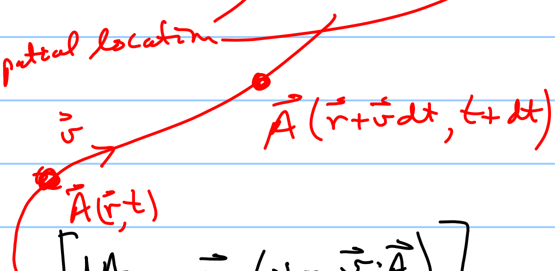
$$\frac{d\vec A}{dt} = \frac{\partial A}{\partial t} + v_x\frac{\partial \vec A}{\partial x} + v_y\frac{\partial \vec A}{\partial y} + v_z\frac{\partial \vec A}{\partial z}$$

$$\frac{d\vec A}{dt} = \left(\vec A(\vec r + \vec v\,dt,\ t+dt) - A(\vec r, t)\right)/dt$$

$$= \frac{\partial \vec A}{\partial x}\Big|_{y,z,t} \overset{\frac{dx}{dt}}{v_x}\frac{dt}{dt} + \frac{\partial \vec A}{\partial y}\Big|_{x,z,t} v_y\frac{dt}{dt} + \frac{\partial \vec A}{\partial z}\Big| v_z\frac{dt}{dt} + \frac{\partial A}{\partial t}\Big|_{x,y,z}\frac{dt}{dt}$$

change spatial location

changing time



$$\frac{d\vec p}{dt} = -q\left[\frac{dA}{dt} + \vec\nabla(V - \vec v\cdot\vec A)\right]$$

$$\frac{d}{dt}\underbrace{(p - qA)}_{} = -\vec\nabla\left[\underbrace{q(V - \vec v\cdot\vec A)}_{\text{vel. dep potential}}\right]$$

$$\frac{dP_{can}}{dt} = -\nabla U$$

$P_{can} = \vec p - q\vec A$

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