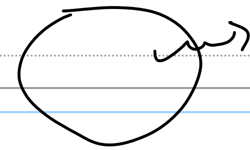
Note Title 6/21/2006



Poynting Theorem

mechanical energy

① $$\frac{\partial W}{\partial t} = -\frac{\partial}{\partial t}\int u\, d\tau - \oint \vec{S}\cdot d\vec{a}$$ integral form

EM energy density

source

$\vec{F}\cdot\vec{v}$

$= q\vec{E} + q\vec{v}\times\vec{B}$

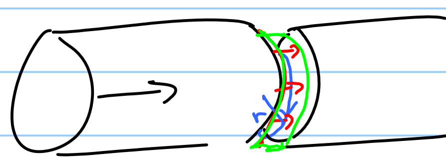
② $$\frac{\partial}{\partial t}\underbrace{\left(u_{EM} + u_{mech}\right)}_{u_{tot}} = -\vec{\nabla}\cdot\vec{S}$$ differential form

$W_{nc} = \Delta(KE + PE)$

thermal

Problem 8.2

$\frac{dQ}{dt} = I_0$



$Q_{plates} = I_0 t$

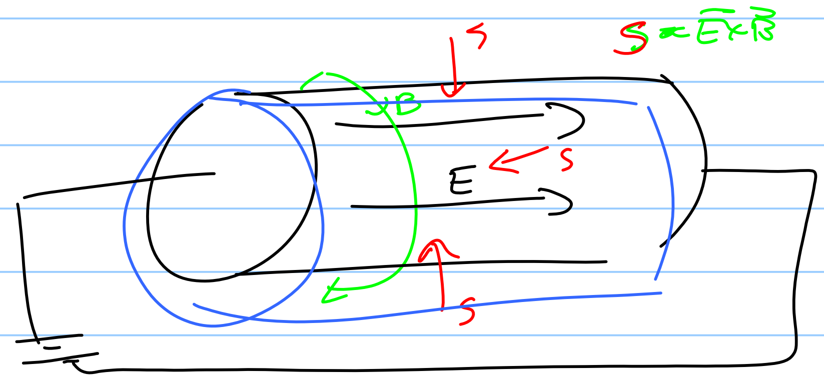
$$E = \frac{\sigma}{\epsilon_0} \qquad \sigma = \frac{Q}{\pi a^2} = \frac{I_0 t}{\pi a^2} \qquad E = \frac{I_0 t}{\epsilon_0 \pi a^2}$$

$$\vec\nabla \times \vec E = -\frac{\partial \vec B}{\partial t} \qquad \vec\nabla \times \vec B = \mu_0 \cancel{\vec J}^{\,0} + \mu_0 \epsilon_0 \frac{\partial \vec E}{\partial t}$$

$$\int \vec\nabla \times \vec B \cdot d\vec a = \oint \vec B \cdot d\vec l = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \int \vec E \cdot d\vec a$$

$$B\, 2\pi r = \mu_0 \epsilon_0 \frac{\partial}{\partial t} E \pi r^2 = \mu_0 \epsilon_0 \pi r^2 \frac{I_0}{\epsilon_0 \pi a^2}$$

$$B = \frac{\mu_0 \epsilon_0 \pi r^{\cancel{2}} I_0}{2\pi \cancel{r}\, \epsilon_0 \pi a^2}$$



steady state

energy is flowing into ohmic wire

$$\frac{\partial W}{\partial t} = -\cancel{\frac{\partial}{\partial t}}\int u\, d\tau - \oint \vec{S}\cdot d\vec{a}$$

$\cancel{}\ 0$

Use Lorentz force & write it as a force per unit volume.
Then use ME to get it in terms of fields.

$$\frac{d}{dt}\vec{P}_{mech} = \vec{F} = \oint \overleftrightarrow{T}\cdot d\vec{a} - \epsilon_0\mu_0 \frac{d}{dt}\int \vec{S}\, d\tau$$

$$\begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix} \begin{bmatrix} da_x \\ da_y \\ da_z \end{bmatrix}$$

flow of momentum (out) surface

rate at which momentum is lost (or gained) within surface

Assume $\underbrace{\vec{F}}_{\text{"}\frac{d\vec{P}}{dt}\text{"}} = 0 = \oint \overleftrightarrow{T}_{EM}\cdot d\vec{a} - \epsilon_0\mu_0 \frac{d}{dt}\int \vec{S}_{EM}\, d\tau$ cons of mom

Looks like $\nabla\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$ $\quad \oint \vec{J}\cdot d\vec{a} = -\frac{\partial}{\partial t} Q_{encl}$

integral form

$$\frac{d\vec{P}_{mech}}{dt} = \oint \overleftrightarrow{T}\cdot d\vec{a} - \epsilon_0\mu_0 \frac{d}{dt}\int \vec{S}_{em}\, d\tau$$

(Poynting thm: flux of energy $\frac{\text{Joules}}{s\, m^2}$

mom density store in fields

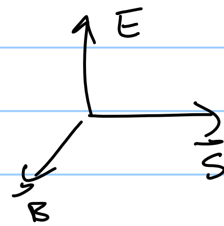
differential form

$$\frac{d}{dt}\int \wp_{mech}\, d\tau = \oint \overleftrightarrow{T}\cdot d\vec{a} - \epsilon_0\mu_0 \frac{d}{dt}\int \wp_{em}\, d\tau$$

divergence theorem

$$\vec{\nabla}\cdot\left(\overleftrightarrow{T}\right) = +\frac{d}{dt}\left(\wp_{em} + \wp_{mech}\right)$$

Ex: plane EM wave

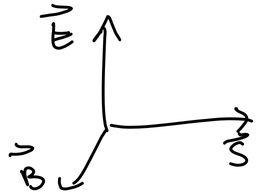


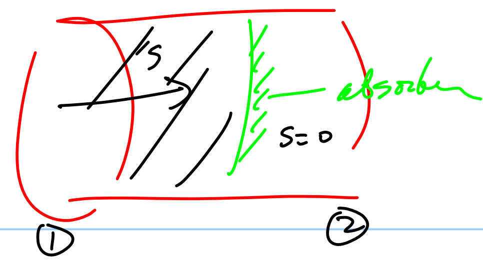
imaginary surface

$$\frac{d}{dt}\int \vec{S}\, d\tau = 0 \quad \text{steady state}$$

$$\oint \overleftrightarrow{T}\cdot d\vec{a} = 0$$

Ex:





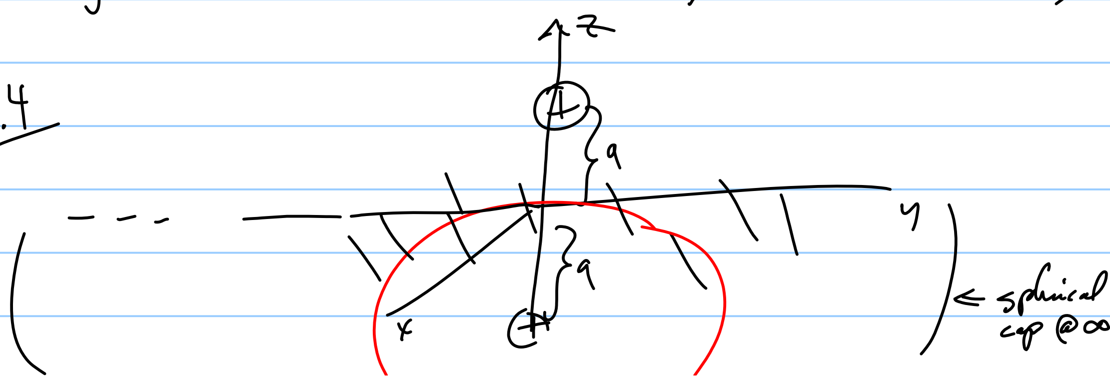
$$\frac{d}{dt}\int \underbrace{\vec S}_{\text{const}}\, d\tau = 0 \quad \text{steady state}$$

$$\oint \overleftrightarrow{T}\cdot d\vec a = \underbrace{\int_{1} \overleftrightarrow{T}\cdot d\vec a}_{0} + \cancel{\int_{2} \overleftrightarrow{T}\cdot d\vec a}$$

$$\text{Force} = \frac{d\vec P_{mech}}{dt} = \underbrace{\oint \overleftrightarrow{T}\cdot d\vec a}_{0} - \epsilon_0\mu_0 \frac{d}{dt}\cancel{\int \vec S_{em}\, d\tau} \quad 0$$

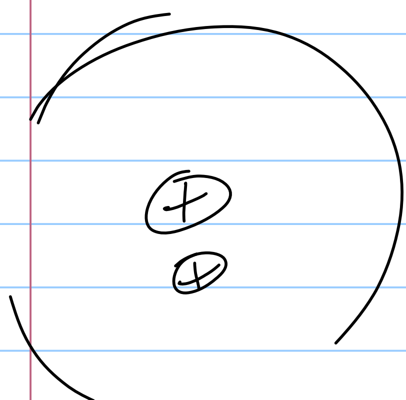
$$T_{ij} = \epsilon_0\left(E_iE_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right)$$

8.4



$$\vec{F} = \frac{d\vec{P}_{mech}}{dt} = \oint \overleftrightarrow{T} \cdot d\vec{a} - \epsilon_0 \mu_0 \frac{d}{dt} \cancel{\int \vec{S}_{em}\, d\tau}^{\,0}$$

T_{11}



$E_1 =$

$$\begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix} \begin{bmatrix} da_x \\ da_y \\ da_z \end{bmatrix}$$

$$T_{ij} = \epsilon_0 \left(E_i E_j - \frac{1}{2} \delta_{ij} E^2 \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2} \delta_{ij} B^2 \right)$$

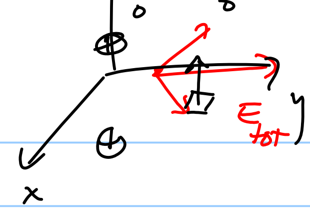
Δz

$$T_{11} = \epsilon_0 \left(E_1 E_1 - \frac{1}{2} \delta_{11} E^2 \right) + 0$$

$E_1 = E_x \qquad E_2 = E_y \qquad E_3 = E_z$

$$\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T & T & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ da_z \end{pmatrix}$$

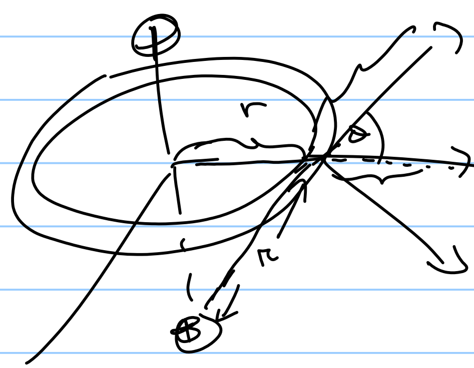
$$d\vec{a} = \underset{= 0}{da_x}\,\hat{x} + \underset{= 0}{da_y}\,\hat{y} + da_z\,\hat{z}$$



$$(E_z)_{tot} = 0$$

Looking for force in z direction

$$F_z = \int T_{zz}\, da_z = \int \epsilon_0 \left(\underbrace{E_{z\,tot}^2}_{0} - \frac{1}{2} E_{tot}^2 \right) da_z$$



$$2 \frac{1}{4\pi\epsilon_0} \frac{q}{\rho^2} \hat{r} \cos\theta$$

$$\cos\theta = \frac{r}{\rho}$$

$$da_z = 2\pi r\, dr$$