

Background

units SI vs Gaussian (cgs)

why Gaussian?

- no ϵ_0, μ_0
- factors of $4\pi, c$ appear in logical place
- E, B fields are in same units, relativistically equivalent
- much literature in Gaussians; must know both why not Gaussian, why SI?
- SI is more convenient for numerical, experimental evaluation.

Force law (cgs)

$$\vec{F}_{12} = \frac{q_1 q_2}{r^2} \hat{e}_r \rightarrow \frac{e^2}{r^2}$$

SI

$$= \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

think of dimensions in terms of defining eqns.

- expressions are often in groups

i. $e^2/r^2 \rightarrow$ force in cgs

main tool for translations

$$e^2 \rightarrow e^2/4\pi\epsilon_0$$

Appx D: numerical conversion to gaussian (not recommended)

Appx E: conversion in terms of equations (better)

convert expression to SI, then evaluate numbers

Notation

Hestel/Marion

$$\text{grad} \rightarrow \vec{\nabla}$$

$$\text{div} \rightarrow \vec{\nabla} \cdot$$

$$\text{curl} \rightarrow \vec{\nabla} \times$$

old fashioned, but emphasizes physical meaning.

Maxwell's equations (vacuum)

SI

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$$

Gaussian

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

Faraday

$$\vec{\nabla} \times \vec{E} = - \frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \times \vec{E} = - \frac{1}{c} \frac{d\vec{B}}{dt}$$

Ampère
corrected

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{d\vec{E}}{dt} + \mu_0 \vec{J}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c} \frac{d\vec{E}}{dt} + \frac{4\pi}{c} \vec{J}$$

$$\vec{D} = \epsilon_0 \vec{E}, \vec{E} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{D} = \epsilon \vec{E} = \vec{E} + 4\pi \vec{P}$$

$$\vec{H} = \frac{1}{\mu_0 \epsilon_0} \vec{B} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{H} = \frac{1}{\mu_0} \vec{B} = \vec{B} - 4\pi \vec{m}$$

Review of Maxwell eqns. (HR, Melia)

Coulomb's law

$$\vec{E} = \frac{q' e_r}{r^2}$$

q' = source charge
 \vec{E} = force/unit charge

EM is a field theory

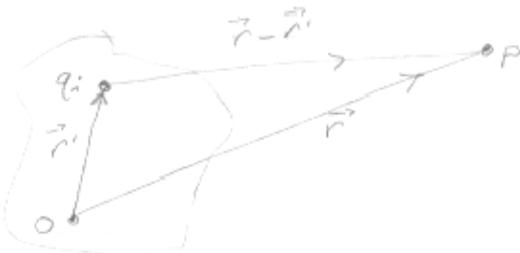
- the field itself has its own reality:

$|E|^2, |B|^2 \propto$ energy density or pressure
 linear, angular momentum.

- Max. Eq \rightarrow fields created by charges, Lorentz force \rightarrow charge

EM field is linear: fields of diff't source pts add. response to fields

$$\vec{E}(\vec{r}) = \sum_i \frac{q(r_i)}{|\vec{r} - \vec{r}_i|^2} \hat{e}_{r-r_i}$$



Electric Flux

$$\Phi_E = \iint_S \vec{E} \cdot d\vec{a}$$

Gauss' law

$$\iint_S \vec{E} \cdot d\vec{a} = \frac{q_{\text{enc}}}{4\pi} \quad \begin{array}{l} \text{e.g. } q = e \text{ at origin} \\ \text{from } 4\pi \text{ sr} \end{array}$$

Surface = sphere, radius = R

$$\iint_S \vec{E} \cdot d\vec{a} = \iint_S \frac{e}{R^2} \cdot R^2 \cdot d\Omega = 4\pi e$$

differential form:

divergence form

read appx A for vector calc
and/or div, grad, curl book.

$$\oint_S \vec{E} \cdot d\vec{a} = \int_V \vec{\nabla} \cdot \vec{E} dv$$

w/ $\int_V \rho dv = q_{\text{enc}}$

$$\rightarrow \vec{\nabla} \cdot \vec{E} = 4\pi\rho$$

since Coulomb field is conservative, i.e.

$$\text{Work} = \oint e \vec{E} \cdot d\vec{l} = 0$$

by Stokes thm $\oint \vec{E} \cdot d\vec{l} = 0 \rightarrow \vec{\nabla} \times \vec{E} = 0$

true only in static case

i.e. $\vec{\nabla} \times (\vec{\nabla} f) = 0$ always

Since $\vec{\nabla} \times \vec{E} = 0 \rightarrow$ write E in terms of a potential

$$\phi(\vec{r}) = - \oint_{C_0} \vec{E} \cdot d\vec{l} \quad \text{(like pot. energy/chg.)}$$

close loop $\rightarrow 0$

∴ inverse

$$\vec{E} = -\vec{\nabla}\phi$$

and Poisson's eqn reads

$$\vec{\nabla} \cdot \vec{\nabla}\phi = \nabla^2\phi = -4\pi\rho$$

Faraday's Law

EMF in a circuit induced by a changing magnetic flux

$$\text{EMF} = -\frac{1}{c} \frac{d\Phi_m}{dt}$$

→ from Gauss units.

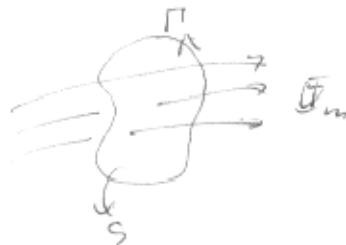
This links the E and B fields:

$$\text{EMF} = \oint_{\Gamma} \vec{E} \cdot d\vec{l} \quad (\text{normally } = 0 \text{ b/c static field is conservative})$$

$$\Phi_m = \int_S \vec{B} \cdot \vec{n} da$$

circulation

$$\rightarrow \oint_{\Gamma} \vec{E} \cdot d\vec{l} = -\frac{1}{c} \int \frac{\partial \vec{B}}{\partial t} \cdot \vec{n} da$$



by Stokes theorem:

$$\rightarrow \int_S (\nabla \times \vec{E}) \cdot \vec{n} da$$

$$\text{i.e. } \nabla \times \vec{E} = \lim_{a \rightarrow 0} \frac{1}{a} \oint_{\Gamma} \vec{E} \cdot d\vec{l}$$

$$\text{and } \nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

diff' form of Faraday.

valid in general for EM fields not just circuits

Conservation of charge + continuity eqn.

current flowing through a surface is

$$I = \oint_S \vec{J} \cdot d\vec{a} = \oint_S \vec{J} \cdot \hat{n} da$$



for a closed surface

$$I_{out} = \oint_S \vec{J} \cdot \hat{n} da = -\frac{dq}{dt}$$

↓

b/c chg is leaving

$$= -\frac{d}{dt} \oint_V \rho dv$$

now use div. thm

$$\oint_V \nabla \cdot \vec{J} dv = - \oint_V \frac{\partial \rho}{\partial t} dv \quad \text{for } S \text{ fixed in space.}$$

$$\rightarrow \nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \quad \text{continuity eqn.}$$

charge is absolutely conserved,

in plasmas often keep track of ion ρ_i and elec ρ_e separately.

$$\rightarrow \nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} < W = \text{ionization rate, source term.}$$

Magnetic field:

no magnetic monopoles proven, so

$$\oint_S \vec{B} \cdot d\vec{a} = 0 \rightarrow \nabla \cdot \vec{B} = 0$$

Lorentz Force

$$\vec{F} = q(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$$

note $\frac{q}{c}$ factor highlights rel. strength of interaction of E, B on a charge.

for uniform velo \rightarrow currents

Biot-Savart

$$\vec{B} = \frac{1}{c} \frac{Q \vec{v} \times \vec{r}}{r^2}$$

single chg Q moving w/ vel \vec{v}

for a current,

$$\vec{B} = \frac{1}{c} \oint \frac{I d\vec{l} \times \vec{r}}{r^2}$$

path along wire

Ampere's law

$$\oint_B \vec{B} \cdot d\vec{l} = \frac{4\pi}{c} I_{\text{link}}$$

\square abt. closed path

$I_{\text{link}} = \text{all currents}$

linking that loop

$$\oint_S \nabla \times \vec{B} \cdot d\vec{a} = \frac{4\pi}{c} \oint_S \vec{J} \cdot d\vec{a}$$

J is total current

$$\nabla \times \vec{B} = \frac{4\pi}{c} J$$

density.