Reflection from metals

review boundary conditions

$\nabla \cdot \vec{D} = 4\pi\rho_f \rightarrow \underline{\varepsilon_1 E_1^\perp - \varepsilon_2 E_2^\perp = 4\pi\rho_{sf}}$

free surface charge

$\nabla \cdot \vec{B} = 0 \rightarrow \underline{B_1^\perp - B_2^\perp = 0}$

$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \rightarrow \underline{E_1^\parallel - E_2^\parallel = 0}$

$\nabla \times \vec{H} = +\frac{1}{c}\frac{\partial \vec{D}}{\partial t} + \frac{4\pi}{c}\vec{J} \rightarrow H_1^\parallel - H_2^\parallel = \underbrace{\vec{J}_{sf} \times \hat{n}}_{\text{free surface current}}$

but if $\vec{J} = \sigma\vec{E}$

a surface current implies a jump in $\vec{E}^\parallel$

$\vec{J} = \vec{J}_{sf}(x,y)\,\delta(z)$

$d \to 0$

which violates continuity of $\vec{E}^\parallel$

$\therefore$ no surface currents.

· Also ignore surface charge too as before.

$\therefore$ B.C. are as in dielectrics, we can use + extend solutions derived earlier w/ complex index

normal incidence

$$E_1^0 = \underbrace{\frac{\tilde{n}_2 - n_1}{\tilde{n}_2 + n_1}}_{\tilde{r}} E_0^0 \qquad \text{reflection}$$

$$E_2^0 = \underbrace{\frac{2n_1}{\tilde{n}_2 + n_1}}_{\tilde{t}} E_0^0 \qquad \text{transmitted}$$

note $\tilde{r}, \tilde{t}$ are complex → phase shifts.

limit high σ

$$\tilde{n}_2 = \sqrt{\epsilon_2 + i\frac{4\pi\sigma_2}{\omega}} \doteq \sqrt{i\frac{4\pi\sigma_2}{\omega}}$$

remember $\delta = \dfrac{c}{\sqrt{2\pi\sigma\omega}}$ = skin depth. $\sqrt{2\pi\sigma} = \dfrac{c}{\delta\sqrt{\omega}}$

$$\rightarrow \quad \tilde{n}_2 = \frac{(1+i)}{\sqrt{2}}\sqrt{\frac{4\pi\sigma_2}{\omega}} = \frac{(1+i)\,c}{\delta\;\omega} = \frac{1+i}{k_{vac}\delta}$$

now calc $\tilde{r}$

$$\tilde{r} = \frac{1+i - n_1 k_{vac}\delta}{1+i + n_1 k_{vac}\delta}$$

→ sign change by our def'n

for $\delta \ll \lambda/n_1$, $\tilde{r}$ close to 1 $\quad |1+i|^2 = 2$

$$\left.\begin{aligned}\tilde{r} &\approx \left(\frac{1+i}{\sqrt{2}} - \frac{n_1 k_{vac}\delta}{\sqrt{2}}\right)^2 \\ &\approx \frac{1+i}{\sqrt{2}} - \frac{2n_1 k_{vac}\delta}{\sqrt{2}}\end{aligned}\right\} \text{skip}$$

$$|\tilde{r}|^2 = R \approx \tfrac{1}{2}(1-2n_1k_{vac}\delta)^2 + \tfrac{1}{2}$$
$$\approx 1 - 2n_1 k_{vac}\delta \quad \rightarrow \text{small losses}$$

$$\cancel{\text{phase} \quad \tan\phi = \frac{\text{Im}\,\tilde{r}}{\text{Re}\,\tilde{r}} = \frac{1/\sqrt{2}}{\frac{1}{\sqrt{2}}(1-2n_1k_{vac}\delta)} \approx 1 + 2n_1k_{vac}\delta}$$

Oblique incidence onto metal surface.

- continue to use Fresnel equations
- be careful with sign conventions
  - best to just force $r \to -1$ at $\theta = 0°$ for both $r_\perp$ and $r_\parallel$
- variable phase shifts with θ, and polarization
  - reflectivity is always best with "s"

wave inside metal:

use Fresnel eq. to get ampl, phase.

internal wave is then

$$s: \quad E_0^0 \, t_\parallel \, e^{i(k_x x + k_z z - \omega t)}$$

now $k_x x \to k_{vac} n_1 \sin\theta_0 \cdot x$ is still the same. (just as in TIR)

$k_z \to$ complex (not pure imag)

$Re(k_z) \to$ osc. portion.

$\therefore$ loss is only in z direction.

phase fronts are in direction of $Re(\vec{k})$:

$$k_x \hat{x} + Re(k_z)\hat{z}$$

in general, this is different than Snell's law

Guided waves.

guiding by metal, TIR, refractive index gradient, photonic band gap / Bragg diffraction.

applications

RF: accelerators

μwave: metal "pipes"

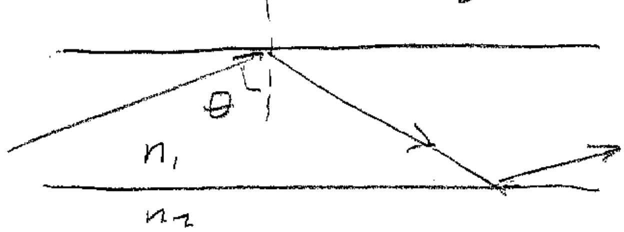
optical: fibers, integrated optics, photonic crystals.

x-ray: capillary tapers.

dynamic w.g.: plasma, self-focusing ...

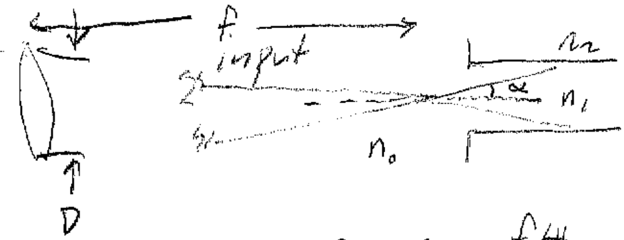
distinguishing features:

$\lambda/a \ll 1 \rightarrow$ "light pipe" guiding, no mode structure



if dielectric n_1 = core n_2 = cladding

$\sin\theta_c = n_2/n_1 \qquad n_1 > n_2$



cone: f/# f/D

num. aperture: $\underline{N.A. \equiv n_0 \sin\alpha_{max}}$

rays $\rightarrow \vec{k} = \hat{x}k_x + \hat{y}k_y + \hat{z}k_z$

θ will connect to modes

guiding condition $\rightarrow$ # guided modes