Fourier Transforms: Transform pairs, theorems in t, ω domain

Definitions and theorems (in Mathematica, use FourierParameters->{1,1}:

Forward transform: $\Im\{f(t)\} \equiv F(\omega) = \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$

Inverse transform: $\Im^{-1}\{F(\omega)\} = f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{-i\omega t} d\omega$

Shift Theorem: $\Im\{f(t-t_0)\} = \exp(+i\omega t_0) F(\omega)$ $\quad \Im^{-1}\{F(\omega-\omega_0)\} = \exp(-i\omega_0 t) f(t)$

Scale Theorem: $\Im\{f(at)\} = \frac{1}{|a|} F(\omega/a)$ $\quad \Im^{-1}\{F(b\omega)\} = \frac{1}{|b|} f(t/b)$

Conjugate: $\Im\{f^*(t)\} = F^*(-\omega)$

Inverse transform pair: $\Im\{F(t)\} = 2\pi\, f(-\omega)$ $\quad \Im^{-1}\{f(\omega)\} = \frac{1}{2\pi} F(-t)$

Convolution: $h(t) = f(t) \otimes g(t) = \int_{-\infty}^{\infty} f(t') g(t-t') dt'$

Convolution w/delta fcn: $\delta(t-t_0) \otimes f(t) = f(t-t_0)$

Convolution theorem:

$f(t) \otimes g(t) = \Im^{-1}\{F(\omega) G(\omega)\}$ $\quad \Im\{f(t) g(t)\} = \frac{1}{2\pi} F(\omega) \otimes G(\omega)$

Parseval's theorem (conservation of energy): $\int |f(t)|^2 dt = \frac{1}{2\pi} \int |F(\omega)|^2 d\omega$

Derivative: $\Im\left\{\frac{\partial^n}{\partial t^n} f(t)\right\} = (-i\omega)^n F(\omega)$

Transform pairs:

Delta functions:

$$\Im\{e^{\pm i\omega_0 t}\} = \int_{-\infty}^{\infty} e^{\pm i\omega_0 t} e^{i\omega t} dt = 2\pi\delta(\omega \pm \omega_0) \qquad \Im^{-1}\{e^{\pm i\omega t_0}\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{\pm i\omega_0 t} e^{-i\omega t} dt = \delta(t \mp t_0)$$

Gaussian: $\Im\{\exp(-t^2/t_w^2)\} = \sqrt{\pi\, t_w^2} \exp(-t_w^2 \omega^2/4)$

Rect function (rect(u) = 1 for $-½ < u < ½$): $\Im\{\mathrm{rect}(t/t_0)\} = t_0\, \mathrm{sinc}(\omega t_0/2)$

$\Im\{\mathrm{sinc}(t/2t_0)\} = 2\pi t_0 rect(\omega t_0)$

Cosine function: $\Im\{\cos(\omega_0 t)\} = \pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$

Array $comb(t/t_0) \equiv \sum_{n=-\infty}^{\infty} \delta(t - nt_0)$: $\Im\{comb(t/t_0)\} = (2\pi/t_0) comb[\omega/(2\pi/t_0)]$