MACS 332 - June 29, 2006 **Exam I - 50 minutes**	**NAME:**

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. Please enclose your final answers in boxes.

1. (10 points)
   Given the following linear system,

$$\begin{aligned} 3x_1 - 9x_2 + 6x_3 &= 0 \qquad (1) \\ -x_1 + 3x_2 - 2x_3 &= 0. \qquad (2) \end{aligned}$$

   a. Determine the general solution set.
   b. What geometric object does the general solution set correspond to?

2. (10 points) Conceptual questions. Briefly respond to the following statements.

- Systems of linear equations have three possible solution sets.
  a. Describe all three.
  b. Give a geometric example for each one.

- Suppose none of the vectors in the set $S = \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$, $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$, is a multiple of one of the other vectors. Is S a linearly independent set? Justify your response.

3. (10 points) Do the following vectors span $\mathbb{R}^3$. Justify your answer.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$$

4. (10 points) Assuming that A is non-singular, prove that $(\mathbf{A}^{\mathrm{T}})^{-1} = (\mathbf{A}^{-1})^{\mathrm{T}}$.

5. (10 points) Find the inverse of the following matrix **and** check your result with an appropriate matrix product.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$