Lecture 4

Note Title 1/18/2006

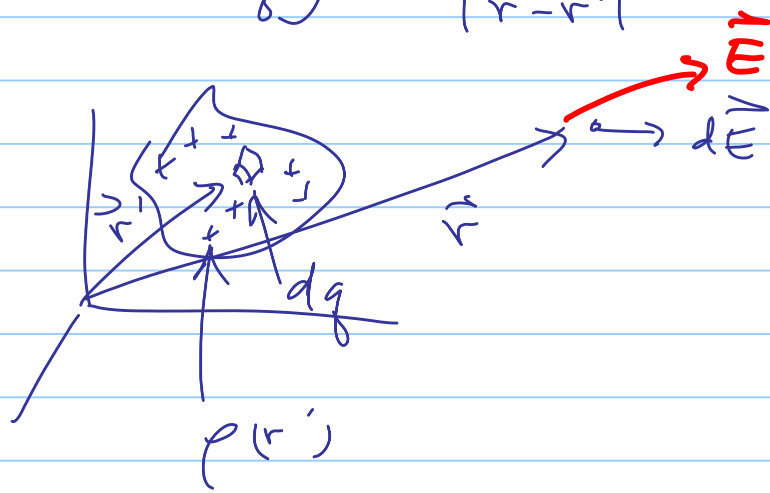
find $\vec{E}$, why?

$$\vec{F} = m\vec{a}$$

"

$$q\vec{E} = q\int d\vec{E} = q\int k\frac{dQ}{r^2}\hat{r}$$

$$= q\frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')\,d^3r'(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} = m\frac{d^2\vec{r}}{dt^2}$$



$$dq = \rho(r)\underbrace{d^3r'}_{dx'dy'dz'}$$

2 ways to find E!

$\int d\vec{E}$

Gauss's Law $\underbrace{\oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}}_{\text{integral form}}$

we need to look differential form of Gauss's law

divergence Th.

$$\int \vec{\nabla}\cdot\vec{F}\, d\tau = \oint \vec{F}\cdot d\vec{a}$$

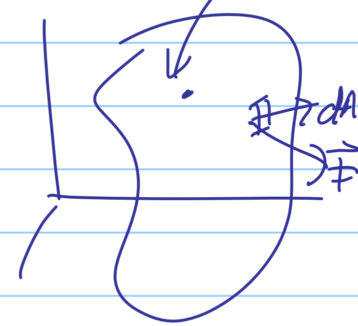
values at bndry

1-D $\int_i^f \frac{dF}{dx}\, dx = F_f - F_i$

3-D $\int \underbrace{\vec{\nabla}\cdot\vec{F}}_{\text{over volume}}\, d\tau = \oint \vec{F}\cdot d\vec{a}$

surface bounds this volume

got # for $\vec{\nabla}\cdot\vec{F}$

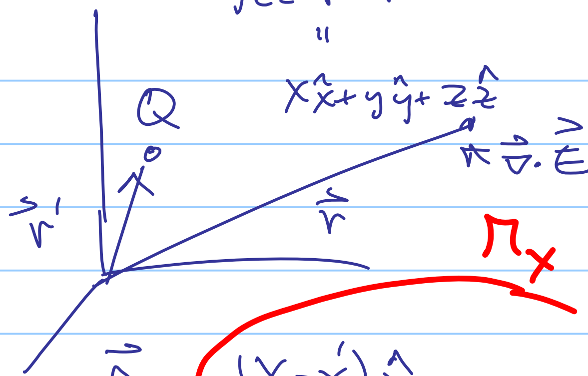


$$\vec{\nabla}\cdot\vec{E} = \frac{1}{4\pi\epsilon_0}\,\vec{\nabla}\cdot\left(Q\,\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{\partial}{\partial x}\mathscr{r}_x + \frac{\partial}{\partial y}\mathscr{r}_y + \frac{\partial}{\partial z}\mathscr{r}_z\right]$$

$$= 0 = \vec{\nabla}\cdot\vec{E}$$

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,' \qquad \vec{r}\,' = x'\hat{x} + y'\hat{y} + z'\hat{z} \qquad \vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$



$\mathscr{r}_x$

$$\frac{\vec{\mathscr{r}}}{|\mathscr{r}|^3} = \frac{(x-x')\,\hat{x}}{\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{3/2}} + \frac{(y-y')\,\hat{y}}{[\;]^{3/2}} + \frac{(z-z')\,\hat{z}}{[\;]^{3/2}}$$

$$\underset{\overset{\shortparallel}{0}}{\int \vec{\nabla}\cdot\vec{E}\,d\tau} = \oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$$

$$\frac{q}{4\pi\epsilon_0}\int \underbrace{\vec{\nabla}\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)}_{4\pi\,\delta^3(\vec{r}-\vec{r}\,')}\,\underset{dx\,dy\,dz}{d\tau}$$

$$4\pi\,\delta^3(\vec{r}-\vec{r}\,') = 4\pi\,\delta(x-x')\,\delta(y-y')\,\delta(z-z')$$

$$\int \vec{\nabla} \cdot \vec{E}\, d\tau = \oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0} = \frac{\int \rho\, d\tau}{\epsilon_0} \quad Q_{enc}$$

$$\Rightarrow \quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$