

Gauss's law:

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$E 4\pi r^2$ $Q_{\text{enc}} = \iiint A r^2 \rho \sin\theta d\theta dr$

Ohm's Law $\vec{J} = \sigma \vec{E}$

$$\nabla \cdot \vec{J} = - \frac{\partial \rho}{\partial t} \quad \rightarrow \quad \oint \vec{J} \cdot d\vec{a} = - \frac{\partial Q_{\text{enc}}}{\partial t}$$

outflow of current
 due to loss of charge
 inside surface

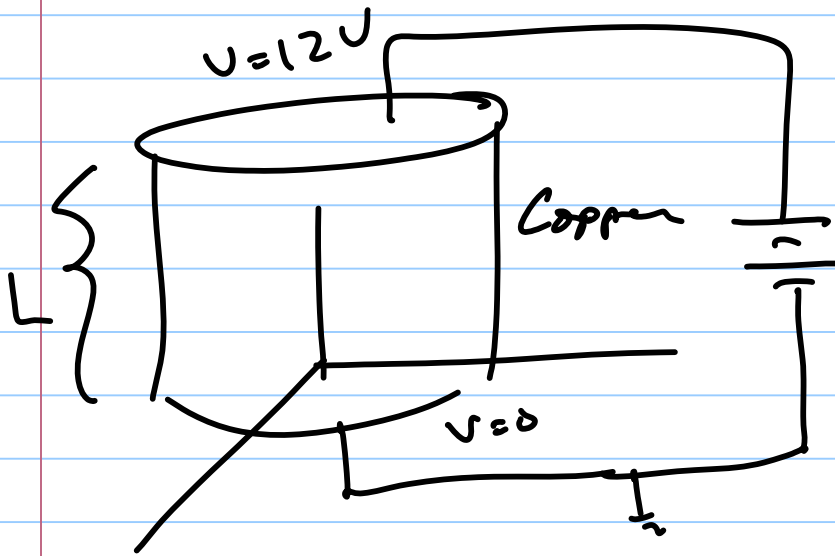
Steady state
 $\nabla \cdot \vec{J} = 0$

$$\vec{\nabla} \cdot \vec{J} = \vec{\nabla} \cdot \underset{\substack{\uparrow \\ \text{const}}}{\sigma \vec{E}} = \sigma \vec{\nabla} \cdot \vec{E} = \phi$$

Steady state

$$\vec{E} = -\vec{\nabla} V \quad \sigma \nabla^2 V = 0 \quad \boxed{\nabla^2 V = 0}$$

Laplace's Eqn



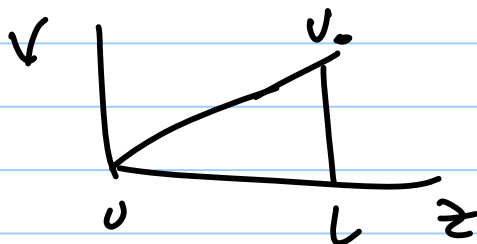
$\nabla^2 V = 0$ solve with boundary cond to find V then get $\vec{E} = -\vec{\nabla} V$
 $\vec{J} = \sigma \vec{E}$ know J

$$\nabla^2 V = 0 \quad \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial V}{\partial s} \right) + \frac{1}{s} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$V = S(s) \underset{\substack{\uparrow \\ \text{const}}}{\phi(\phi)} z, \phi$$

$$V(s) = \phi \quad V(\phi) = \phi$$

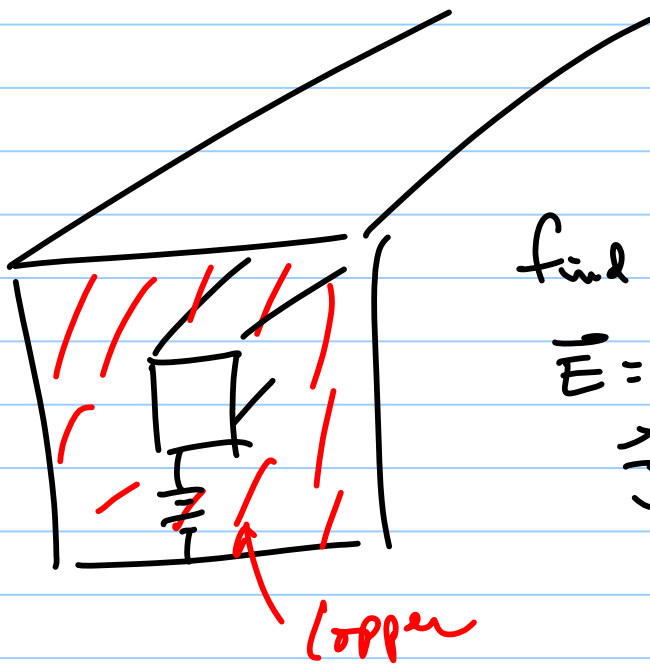
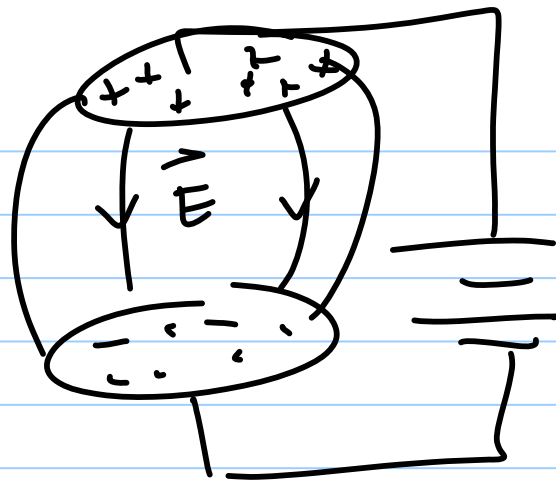
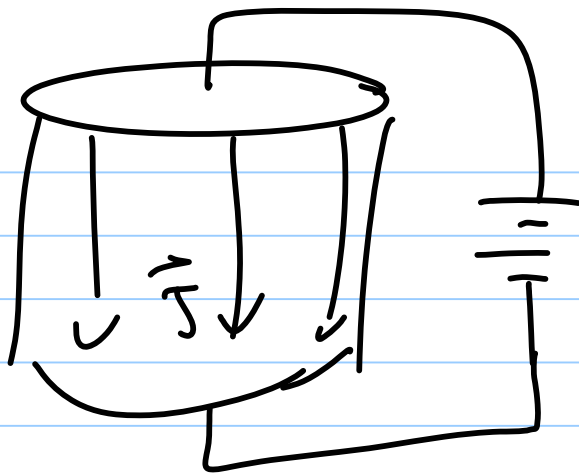
$$\frac{\partial^2 z}{\partial z^2} = 0 \quad z = Az + B$$



$$\vec{E} = -\vec{\nabla} z = \text{const}$$

\vec{J} is const

$$\vec{J} \propto \vec{E}$$



find V using Relaxation method

$$\vec{E} = -\nabla V \text{ then}$$

$$\vec{J} = \sigma \vec{E}$$

