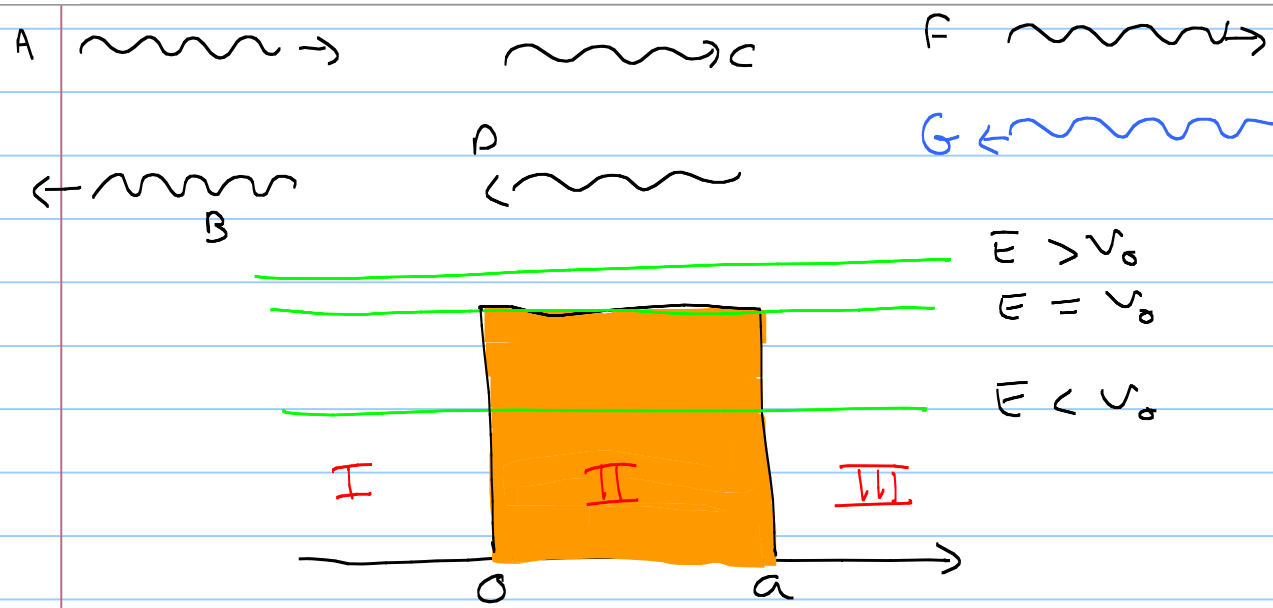
3_3_08

Note Title 3/3/2008



I $\quad -\dfrac{\hbar^2}{2m}\psi''(x) = E\psi(x)$ $\quad\Big|\quad \psi'' + k_1^2\psi = 0$

$k_1^2 = 2mE/\hbar^2$

II $\quad -\dfrac{\hbar^2}{2m}\psi'' + V_0\psi = E\psi$ $\quad\Big|\quad \psi'' + k_2^2\psi = 0$

$k_2^2 = 2m(E - V_0)$

III $\quad -\dfrac{\hbar^2}{2m}\psi''(x) = E\psi(x)$ $\quad\Big|\quad \psi'' + k_1^2\psi = 0$

Regions I + III solutions to S.E. are oscillatory. in Region II all depens on E versus V_0

Case ① $\quad E = V_0$

$$\psi'' + k_2^2\psi = 0$$

$$k_2^2 = \frac{2m}{\hbar^2}(E - V_0) \Rightarrow \psi'' = 0$$

$$\Rightarrow \boxed{\psi(x) = ax + b}$$

case ② $E > V_0$

$\psi'' + k_2^2\psi = 0$ k_2 real + positive

solutions oscillatory

case ③ $E < V_0$

$$\psi'' + k_2^2\psi = 0 \qquad k_2^2 = \frac{2m}{\hbar^2}(E - V_0)$$

negative

so $k_2 = i/\hbar\sqrt{2m(V_0 - E)} \equiv i\kappa$

κ real

Solution $e^{\pm ik_2x} \Rightarrow e^{\pm\kappa x}$

region II $E < V_0$

$$\boxed{\psi(x) = Ce^{\kappa x} + De^{-\kappa x}}$$

$$T = \left\|\frac{4k_1k_2e^{-ia(k_1-k_2)}}{(k_1+k_2)^2 - e^{2iak_2}(k_1-k_2)^2}\right\|^2$$

Analysis

$E < V_0$ $$\boxed{T = \frac{1}{1 + \frac{V_0^2 \sinh(k_2 a)}{4E(V_0 - E)}}}$$

$E > V_0$ $$\boxed{T = \frac{1}{1 + \frac{V_0^2 \sin(k_2 a)}{4E(E - V_0)}}}$$

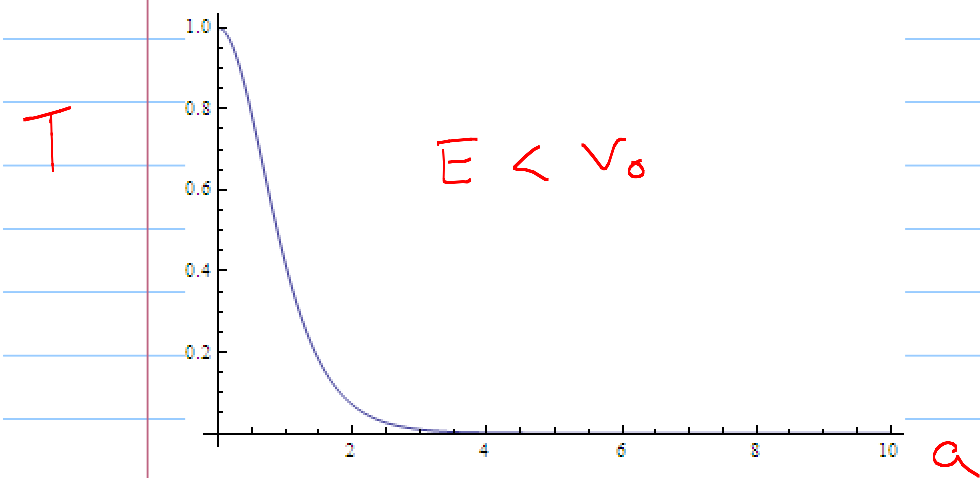
Two nonclassical results:

For $E > V_0$ nonzero R.

For $E < V_0$ nonzero T (tunneling)

For $E = V_0$

$$\boxed{T = \frac{1}{1 + m a^2 V_0 / 2\hbar^2}}$$



Remember in QM, T/R refer to the probability of penetration / reflection