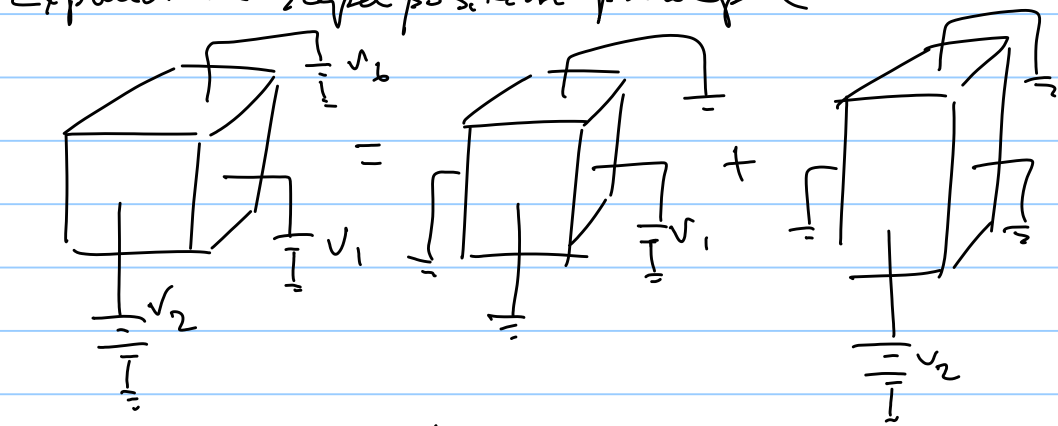
Lecture 14

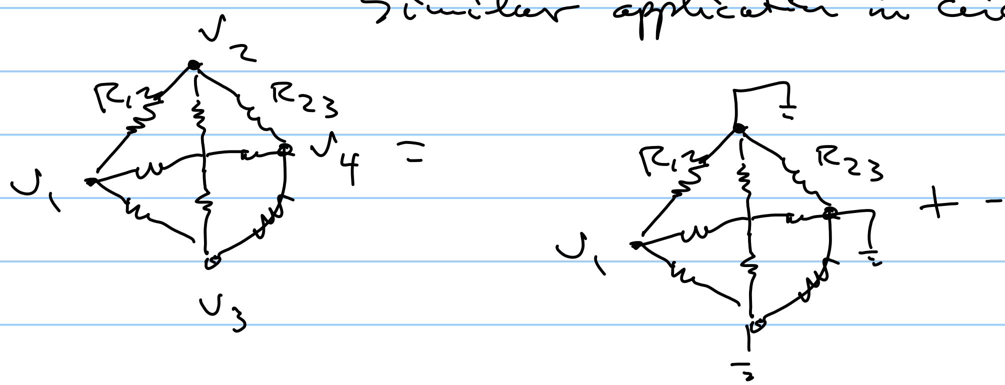
Note Title 2/13/2006

Expand on superposition principle



+ . . .

Similar application in circuits



Spherical coords

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\varphi^2} = 0$$

multiply thru by $r^2\sin^2\theta$ let $V(r,\theta,\varphi) = R(r)\,\Theta(\theta)\,\Phi(\varphi)$

subst. into $\nabla^2 V = 0$ & divide by $V(r,\theta,\varphi)$

$$\underbrace{\frac{\sin^2\theta}{R(r)}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \frac{\sin\theta}{\Theta(\theta)}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)}_{R(r)\,\Theta(\theta)\ =\ f(r,\theta)} + \underbrace{\frac{1}{\Phi(\varphi)}\frac{d^2\Phi}{d\varphi^2}}_{f(\varphi)} = 0$$

$$R(r)\,\Theta(\theta) = f(r,\theta) + f(\varphi) = 0$$

$$m^2 - m^2 = 0$$

$$\frac{d^2\Phi}{d\varphi^2} = -m^2\Phi \qquad \Phi(\varphi) = A\sin(m\varphi) + B\cos(m\varphi)$$

V must be single valued. $\sin m\varphi = \sin m(\varphi + 2\pi)$

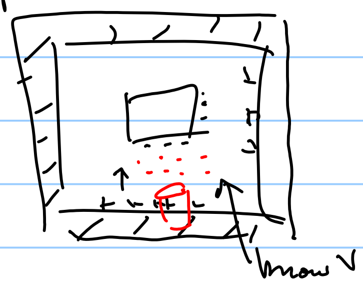
$\Rightarrow$ m must be 0, 1, 2, ...

$$V = f(r,\theta)\left[A\sin m\varphi + B\cos m\varphi\right]$$

Let $m = 0$ in all our problems

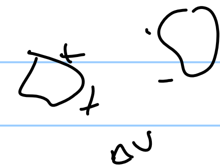
$\Theta(\theta)$ is given by Legendre polys

Capacitance



$\frac{\delta V}{\delta y} = \frac{\Delta V}{\Delta y}$

$C = \frac{Q}{V}$



$\frac{\partial V}{\partial n}\Big|_{above} - \frac{\partial V}{\partial n}\Big|_{below} = \frac{-\sigma}{\epsilon_0}$

Vector $\vec{V} = \sum_{i=1}^{3} a_i \hat{x}_i$ $\quad \hat{x}_1 = \hat{i} = \hat{x}$

$V \cdot \hat{x}_i = a_i$ $\quad \hat{x}_i \cdot \hat{x}_j = \delta_{ij}$ $\quad \hat{x}_2 = \hat{y}$ $\quad \hat{x}_3 = \hat{z}$

$\underline{\psi(x)} = \sum_n a_n \underline{\psi_n(x)}$ Fourier decomp

$$\int \psi_n(x) \psi_m(x)\, dx = \delta_{nm}$$

$$\int \psi(x) \psi_n(x)\, dx = a_n$$

$$|\psi\rangle = \sum_n a_n |\varphi_n\rangle$$

$$\int_0^b \sin\left(\frac{n\pi y}{b}\right)\sin\left(\frac{m\pi y}{b}\right) dy = \frac{b}{2}\delta_{nm}$$

$$\int_{-1}^{1} P_\ell(x)\, P_m(x)\, dx = \int_0^\pi P_\ell(\cos\theta)\, P_m(\cos\theta) \sin\theta\, d\theta =$$

$x = \cos\theta$

$$= \begin{cases} 0 & \text{if } \ell \neq m \\ \frac{2}{2m+1} & \text{if } \ell = m \end{cases}$$