3-1.

a) $$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10^4 \text{ dyne/cm}}{10^2 \text{ gram}}} = \frac{10}{2\pi}\sqrt{\frac{\frac{\text{gram}\cdot\text{cm}}{\text{sec}^2\cdot\text{cm}}}{\text{gram}}} = \frac{10}{2\pi}\text{ sec}^{-1}$$

or,

$$\boxed{\nu_0 \cong 1.6 \text{ Hz}} \tag{1}$$

$$\tau_0 = \frac{1}{\nu_0} = \frac{2\pi}{10}\text{ sec}$$

or,

$$\boxed{\tau_0 \cong 0.63 \text{ sec}} \tag{2}$$

b) $$E = \frac{1}{2}kA^2 = \frac{1}{2}\times 10^4 \times 3^2 \text{ dyne-cm}$$

so that

$$\boxed{E = 4.5\times 10^4 \text{ erg}} \tag{3}$$

c) The maximum velocity is attained when the total energy of the oscillator is equal to the kinetic energy. Therefore,

$$\frac{1}{2}mv_{\max}^2 = 4.5\times 10^4 \text{ erg}$$

$$\text{v}_{\max} = \sqrt{\frac{2\times 4.5\times 10^4}{100}}$$

or,

$$\boxed{v_{\max} = 30 \text{ cm/sec}} \tag{4}$$

3-2.

a) The statement that at a certain time $t = t_1$ the maximum amplitude has decreased to one-half the initial value means that

$$\left|x_{en}\right| = A_0 e^{-\beta t_1} = \frac{1}{2}A_0 \tag{1}$$

or,

$$e^{-\beta t_1} = \frac{1}{2} \tag{2}$$

so that

$$\beta = \frac{\ln 2}{t_1} = \frac{0.69}{t_1} \tag{3}$$

Since $t_1 = 10$ sec ,

$$\boxed{\beta = 6.9 \times 10^{-2}\ \text{sec}^{-1}} \tag{4}$$

b) According to Eq. (3.38), the angular frequency is

$$\omega_1 = \sqrt{\omega_0^2 - \beta^2} \tag{5}$$

where, from Problem 3-1, $\omega_0 = 10\ \text{sec}^{-1}$. Therefore,

$$\begin{aligned} \omega_1 &= \sqrt{(10)^2 - \left(6.9 \times 10^{-2}\right)^2} \\ &\cong 10\left[1 - \frac{1}{2}(6.9)^2 \times 10^{-6}\right] \text{sec}^{-1} \end{aligned} \tag{6}$$

so that

$$\boxed{\nu_1 = \frac{10}{2\pi}(1 - 2.40 \times 10^{-5})\ \text{sec}^{-1}} \tag{7}$$

which can be written as

$$\nu_1 = \nu_0(1 - \delta) \tag{8}$$

where

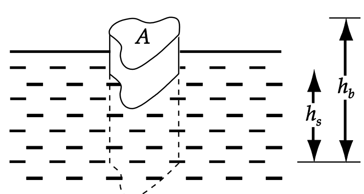
$$\delta = 2.40 \times 10^{-5} \tag{9}$$

That is, ν_1 is only slightly different from ν_0.

c) The *decrement* of the motion is defined to be $e^{\beta\tau_1}$ where $\tau_1 = 1/\nu_1$. Then,

$$\boxed{e^{\beta\tau_1} \simeq 1.0445}$$

3-7.



Let A be the cross-sectional area of the floating body, h_b its height, h_s the height of its submerged part; and let ρ and ρ_0 denote the mass densities of the body and the fluid, respectively.

The volume of displaced fluid is therefore $V = Ah_s$. The mass of the body is $M = \rho A h_b$.

There are two forces acting on the body: that due to gravity (Mg), and that due to the fluid, pushing the body up ($-\rho_0 gV = -\rho_0 gh_s A$).

The equilibrium situation occurs when the total force vanishes:

$$\begin{aligned} 0 &= Mg - \rho_0 gV \\ &= \rho gAh_b - \rho_0 gh_s A \end{aligned} \tag{1}$$

which gives the relation between h_s and h_b:

$$h_s = h_b \frac{\rho}{\rho_0} \tag{2}$$

For a small displacement about the equilibrium position ($h_s \to h_s + x$), (1) becomes

$$M\ddot{x} = \rho Ah_b \ddot{x} = \rho gAh_b - \rho_0 g\left(h_s + x\right)A \tag{3}$$

Upon substitution of (1) into (3), we have

$$\rho Ah_b \ddot{x} = -\rho_0 gxA \tag{4}$$

or,

$$\ddot{x} + g\frac{\rho_0}{\rho h_b} x = 0 \tag{5}$$

Thus, the motion is oscillatory, with an angular frequency

$$\omega^2 = g\frac{\rho_0}{\rho h_b} = \frac{g}{h_s} = \frac{gA}{V} \tag{6}$$

where use has been made of (2), and in the last step we have multiplied and divided by A. The period of the oscillations is, therefore,

$$\boxed{\tau = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{V}{gA}}} \tag{7}$$

Substituting the given values, $\tau \simeq 0.18\ \text{s}$.

3-11. The total energy of a damped oscillator is

$$E(t) = \frac{1}{2} m\dot{x}(t)^2 + \frac{1}{2} kx(t)^2 \tag{1}$$

where

$$x(t) = Ae^{-\beta t}\cos\left(\omega_1 t - \delta\right) \tag{2}$$

$$\dot{x}(t) = Ae^{-\beta t}\left[-\beta\cos\left(\omega_1 t - \delta\right) - \omega_1 \sin\left(\omega_1 t - \delta\right)\right] \tag{3}$$

$$\omega_1 = \sqrt{\omega_0^2 - \beta^2}, \quad \omega_0 = \sqrt{\frac{k}{m}}$$

Substituting (2) and (3) into (1), we have

$$E(t)=\frac{A^2}{2}e^{-2\beta t}\Big[\left(m\beta^2+k\right)\cos^2\left(\omega_1 t-\delta\right)+m\omega_1^2\sin^2\left(\omega_1 t-\delta\right) \\ +2m\beta\omega_1\sin\left(\omega_1 t-\delta\right)\cos\left(\omega_1 t-\delta\right)\Big] \tag{4}$$

Rewriting (4), we find the expression for $E(t)$:

$$\boxed{E(t)=\frac{mA^2}{2}e^{-2\beta t}\left[\beta^2\cos 2\left(\omega_1 t-\delta\right)+\beta\sqrt{\omega_0^2-\beta^2}\sin 2\left(\omega_1 t-\delta\right)+\omega_0^2\right]} \tag{5}$$

Taking the derivative of (5), we find the expression for $\dfrac{dE}{dt}$:

$$\boxed{\frac{dE}{dt}=\frac{mA^2}{2}e^{-2\beta t}\Big[\left(2\beta\omega_0^2-4\beta^3\right)\cos 2\left(\omega_1 t-\delta\right) \\ -4\beta^2\sqrt{\omega_0^2-\beta^2}\sin 2\left(\omega_1 t-\delta\right)_0-2\beta\omega^2\Big]} \tag{6}$$

The above formulas for E and dE/dt reproduce the curves shown in Figure 3-7 of the text. To find the average rate of energy loss for a lightly damped oscillator, let us take $\beta \ll \omega_0$. This means that the oscillator has time to complete some number of periods before its amplitude decreases considerably, i.e. the term $e^{-2\beta t}$ does not change much in the time it takes to complete one period. The cosine and sine terms will average to nearly zero compared to the constant term in dE/dt, and we obtain in this limit

$$\frac{dE}{dt}\simeq -m\beta\omega_0^2A^2e^{-2\beta t}$$

3-13. For the case of critical damping, $\beta=\omega_0$. Therefore, the equation of motion becomes

$$\ddot{x}+2\beta\dot{x}+\beta^2x=0 \tag{1}$$

If we assume a solution of the form

$$x(t)=y(t)e^{-\beta t} \tag{2}$$

we have

$$\left.\begin{aligned}\dot{x}&=\dot{y}e^{-\beta t}-\beta ye^{-\beta t}\\ \ddot{x}&=\ddot{y}e^{-\beta t}-2\beta\dot{y}e^{-\beta t}+\beta^2ye^{-\beta t}\end{aligned}\right] \tag{3}$$

Substituting (3) into (1), we find

$$\ddot{y}e^{-\beta t}-2\beta\dot{y}e^{-\beta t}+\beta^2ye^{-\beta t}+2\beta\dot{y}e^{-\beta t}-2\beta^2ye^{-\beta t}+\beta^2ye^{-\beta t}=0 \tag{4}$$

or,

$$\ddot{y}=0 \tag{5}$$

Therefore,

$$y(t) = A + Bt \tag{6}$$

and

$$\boxed{x(t) = (A + Bt)e^{-\beta t}} \tag{7}$$

which is just Eq. (3.43).

3-14. For the case of overdamped oscillations, $x(t)$ and $\dot{x}(t)$ are expressed by

$$x(t) = e^{-\beta t}\left[A_1 e^{\omega_2 t} + A_2 e^{-\omega_2 t}\right] \tag{1}$$

$$\dot{x}(t)e^{-\beta t}\left[-\beta\left(A_1 e^{\omega_2 t} + + A_2 e^{-\omega_2 t}\right) + \left(A_1\omega_2 e^{\omega_2 t} - A_2\omega_2 e^{-\omega_2 t}\right)\right] \tag{2}$$

where $\omega_2 = \sqrt{\beta^2 - \omega_0^2}$. Hyperbolic functions are defined as

$$\cosh y = \frac{e^y + e^{-y}}{2}, \quad \sinh y = \frac{e^y - e^{-y}}{2} \tag{3}$$

or,

$$\left.\begin{aligned} e^y &= \cosh y + \sinh y \\ e^{-y} &= \cosh y - \sinh y \end{aligned}\right] \tag{4}$$

Using (4) to rewrite (1) and (2), we have

$$\boxed{x(t) = (\cosh \beta t - \sinh \beta t)\left[(A_1 + A_2)\cosh \omega_2 t + (A_1 - A_2)\sinh \omega_2 t\right]} \tag{5}$$

and

$$\boxed{\begin{aligned} \dot{x}(t) = (\cosh \beta t - \sinh \beta t)\Big[&(A_1\omega_2 - A_1\beta)(\cosh \omega_2 t + \sinh \omega_2 t) \\ &-(A_2\beta + A_2\omega_2)(\cosh \omega_2 t - \sinh \omega_2 t)\Big] \end{aligned}} \tag{6}$$