**MATH225, Fall 2008** **Name:**
**Worksheet 6 (3.2-3.5)** **Section:**

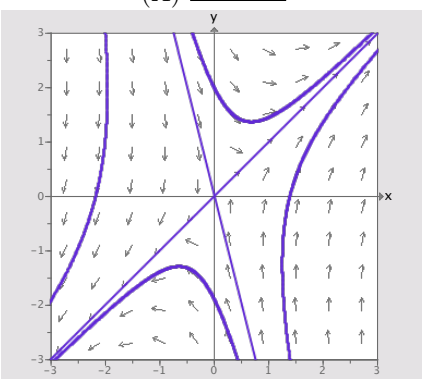
For full credit, you must show all work and box answers.

1. Match each of the following matrices with its possible phase portrait. Hint: Find the eigenvalues.

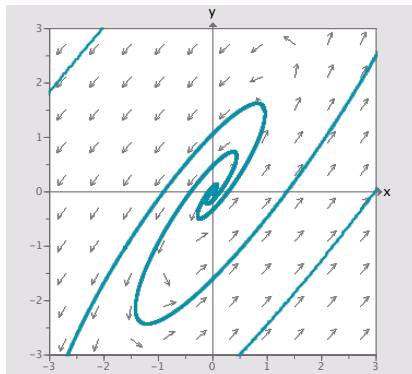
(i) $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$ (ii) $A = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}$

(iii) $A = \begin{pmatrix} -1 & 2 \\ -2 & 4 \end{pmatrix}$ (iv) $A = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}$

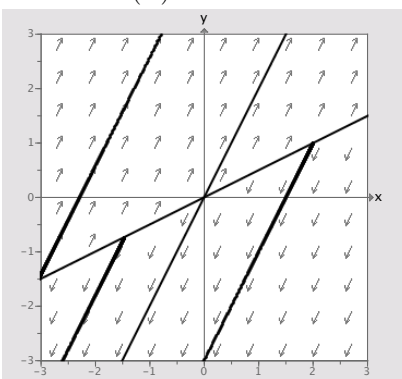
(A) _____



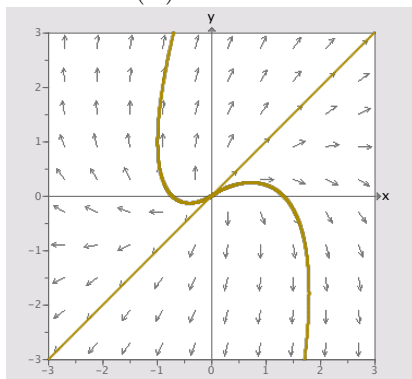

(B) _____




(C) _____




(D) _____



2. Given the system

$$\begin{aligned} \frac{dx}{dt} &= x - 2y \\ \frac{dy}{dt} &= 3x - 4y \end{aligned}$$

(a) Find the general solution.

(b) Sketch the phase portrait. Make sure you include the straight-line solutions and at least two other trajectories. Classify the origin.

(c) Find the particular solution that satisfies the initial condition $(x(0), y(0)) = (1, 0)$. Report your solution as one real vector.

3. Given the system

$$\mathbf{Y}' = \begin{pmatrix} 2 & 8 \\ -1 & -2 \end{pmatrix} \mathbf{Y}$$

(a) Find the general solution.

(b) Classify the origin and graph the phase portrait using HPGSystemSolver (from the software associated with your book). Graph at least three separate trajectories in the phase portrait. Print and include your results.

(c) Find the particular solution that satisfies the initial condition $\mathbf{Y}(0) = (8, 6)$. Report your solution as one real vector.

4. Given the second-order differential equation

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 0$$

(a) Find the corresponding first-order system by letting $v = \dfrac{dy}{dt}$.

(b) Find the general solution.

(c) Sketch the phase portrait. Classify the origin.

(d) What is the natural period of the system?

(e) Find the particular solution that satisfies the initial condition $\mathbf{Y}(0) = (2, -2)$. Report your solution as one real vector.

(f) Sketch the $x(t)$- and $y(t)$-graphs for your particular solution from part (e).

5. Given the system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -1 & 3 \\ -3 & 5 \end{pmatrix} \mathbf{Y}$$

(a) Find the particular solution with the initial condition $\mathbf{Y}(0) = (1, 2)$. Report your solution as one real vector.

(b) Graph the phase portrait using HPGSystemSolver (from the software associated with your book). Graph at least three separate trajectories in the phase portrait. Print and include your results.

6. Given the system

$$\begin{aligned} \frac{dx}{dt} &= 3x + 6y \\ \frac{dy}{dt} &= -x - 2y \end{aligned}$$

(a) Find the general solution.

(b) Sketch the phase portrait by hand or graph the phase portrait using HPGSystemSolver. Graph at least three separate trajectories in the phase portrait. Print and include your results if you used the software.