

Pure State  $\vec{J} = -\frac{\partial \vec{E}}{\partial t}$  see formulae above  
 Mixed State  $\vec{J} = \vec{J}_0 + \vec{J}_1$  vector plus

$$\vec{J}_1 \cdot \vec{S} = -\frac{\partial \vec{E}}{\partial t} \Rightarrow \vec{S} \cdot d\vec{J}_1 = -\frac{\partial \vec{E}}{\partial t} d\vec{u}$$

$$\vec{J}_1 \cdot \vec{S} = -\frac{\partial \vec{E}}{\partial t} \text{ energy density} \quad \vec{S} \cdot d\vec{J}_1 = -\frac{\partial \vec{E}}{\partial t} d\vec{u}$$

Poynting vector + wave intensity

definition  $\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{H}$  assuming real fields

- note that for most materials,  $\mu = 1$  (non-magnetic)  
so in this case  $\vec{H} = \vec{B}$
- in SI  $\vec{S} = \vec{E} \times \vec{H}$   $S = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

represent  $\vec{H}$  ( $\text{or } \vec{B}$ ) in terms of  $\vec{E}$

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \rightarrow \vec{k} \times \vec{E} = \frac{\omega}{c} \vec{B}$$

$$\therefore \vec{S} = \frac{c}{4\pi} \cdot \frac{c}{\omega} \vec{E} \times (\vec{k} \times \vec{E})$$

$$\text{use } \vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

$$\rightarrow \vec{E} \times (\vec{k} \times \vec{E}) = \vec{k}(\vec{E} \cdot \vec{E}) - \vec{E}(\vec{E} \cdot \vec{k})$$

$$\text{Finally, } \vec{S} = \frac{c^2}{4\pi\omega} \vec{k}(\vec{E} \cdot \vec{E}) = \underline{\underline{\frac{cn\hat{k}\vec{E} \cdot \vec{E}}{4\pi}}}$$

since  $\vec{k} = \frac{\omega}{c} \hat{n}$  in a medium

Notes:

- $\vec{S}$  is in direction of  $\vec{k}$
- $\vec{S}$  represents a directional power flux
- use  $\vec{E} \cdot \vec{E}$  to handle polarization states.
- calc. assumes real fields, real  $n$

"intensity" = "irradiance" =  $I = |\vec{S}|$

$$\text{in SI } I = \frac{1}{2} \epsilon_0 c n \vec{E}_0 \cdot \vec{E}_0$$

## Photons + classical EM

Photons carry energy  $h\nu = h\gamma$

Consider a beam of light, "square" pulse

$$\text{area } A \quad \langle \vec{S} \rangle = \frac{1}{2} \epsilon_0 n c |E_0|^2 = I A$$

$$\text{in SI } I = \frac{1}{2} \epsilon_0 n c |E_0|^2 \text{ W/m}^2$$

$I = \text{intensity}$   
(time avg)

Power in beam

$$P = \int I(x, y) dx dy = I \cdot A \text{ Watts}$$

=  $\int \vec{S} \cdot d\vec{a}$  (accounts for any tilt)

Energy in pulse:

$$E = \int P(t) dt = I \cdot A \cdot t$$

Thinking of light as photons,

$$E = N h \nu \quad N = \# \text{ photons in pulse.}$$

Examples

Over visible part of spectrum, solar intensity at Earth's surface is  $\sim 1 \text{ kW/m}^2 = 0.1 \text{ W/cm}^2$

$$h\nu \sim 2 \text{ eV} \sim 3 \times 10^{-19} \text{ J} \rightarrow \sim 3 \times 10^{18} \text{ photons/s/cm}^2$$

a 1 mW laser pointer in 1 mm diameter  $I = 0.13 \text{ W/cm}^2$

- 1 mW avg power, 100 MHz up rate, 10 fs pulse:

$$I = 0.13 \cdot 10^{14} / 10^8 = 1.3 \times 10^5 \text{ W/cm}^2$$

$$10 \mu\text{m spot} \rightarrow I = 1.3 \times 10^9 \text{ W/cm}^2$$

## Photon pressure

$$\text{photon momentum } p = h\nu = \frac{\hbar\omega}{c} = \hbar k$$

use  $\vec{p} = \hbar\vec{k}$

$$\text{pressure} = \text{energy density} = \frac{N\hbar\omega}{A ct} = \frac{I}{c} = \frac{\langle \hat{S} \rangle}{c}$$

examples: reflect @ normal incidence:

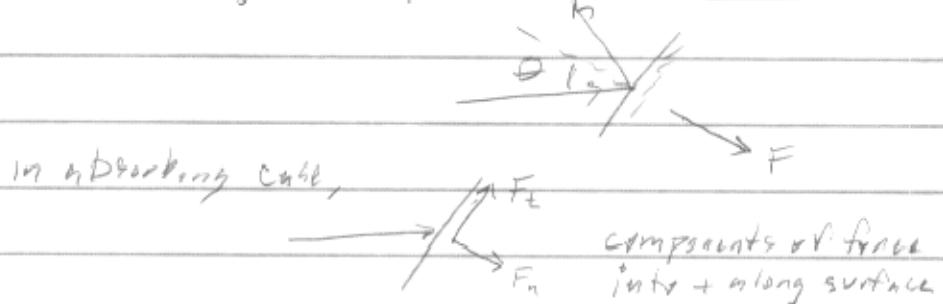


$$\Delta p = \hbar k - (-\hbar k) = 2\hbar k \quad \text{per photon}$$

$$\text{pressure: } 2I/c$$

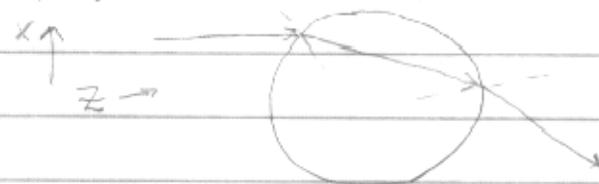
if all is absorbed  $\rightarrow$  pressure =  $I/c$

HW: Show angular dependence is  $\cos^2\theta$



"photon drag" induces a small current in conductors

trapping: refractive sphere



net  $\Delta p$  by redirecting

transverse intensity gradient  $\rightarrow$  push object toward high intensity