Relaxation Time for Acoustic Phonon Scattering

3-15-12

Follows closely from Askerov Section 11.2

We begin with the transition probability for scattering by acoustic phonons

$$W(\vec{K},\vec{K}') = \frac{2\pi}{MN}\frac{E_1^2}{\hbar v_0^2} k_B T\, \delta(\varepsilon_{\vec{K}'} - \varepsilon_{\vec{K}})$$

E_1 - Deformation Potential Constant

M - Atomic Mass

v_0 - Crystal Sound velocity

N - Mean number of phonons

Lets place this expression in the general tau expression to yield

$$\frac{1}{\tau} = \frac{2\pi}{MN}\frac{E_1^2}{\hbar v_0^2} k_B T \sum_{K'} \left(1 - \frac{\vec{K}\cdot\vec{K}'}{K^2}\right) \delta(\varepsilon_{K'} - \varepsilon_K)$$

First we turn the Sum to an integral in spherical coordinates

Note: $\delta[\varepsilon(K)] = \delta(K)\frac{\partial K}{\partial \varepsilon}$

$$\int_0^{\pi}\int_0^{2\pi}\int_0^{\infty} (1-\cos\theta)\, \delta(\varepsilon_{K'} - \varepsilon_K)\, K^2 \sin\phi \, dK'\, d\theta\, d\phi$$

$$\int_0^{\pi}\int_0^{2\pi}\int_0^{\infty} (\sin\phi - \sin\phi\cos\theta)\, \delta(K'-K)\frac{\partial K}{\partial \varepsilon} K^2 \, dK'\, d\theta\, d\phi$$

$$4\pi K^2 \frac{\partial K}{\partial \varepsilon}\int_0^{\infty} \delta(K'-K)\, dK'$$

$$4\pi K^2 \frac{\partial K}{\partial \varepsilon}\cdot\frac{V}{(2\pi)^3} = K^2 V \frac{\partial K}{\partial \varepsilon}\cdot\frac{1}{2\pi^2}$$

Placing this term back in the tau expression

$$\frac{1}{\tau} = \frac{V}{MN}\frac{E_1^2}{\pi\hbar v_0^2}\cdot k_B T\cdot K^2\cdot\frac{\partial K}{\partial \varepsilon}$$