MATH 332 - Linear Algebra June 16, 2009

Homework 1, Summer 2009 **Due**: June 18, 2009

Row Reduction - Echelon Forms - Solutions to Linear Systems

1. Suppose a, b, c, and d are constants such the system

$$ax_1 + bx_2 = 0$$
$$cx_1 + dx_2 = 0$$

with $a \neq 0$. Using row reduction solve for x_1 and x_2 and list any constraints needed, on a, b, c, d, for unique solutions. [1]

2. Given the linear system

$$\begin{aligned} 6x_1 + 18x_2 - 4x_3 &= 20 \\ -x_1 - 3x_2 + 8x_3 &= 4 \\ 5x_1 + 15x_2 - 9x_3 &= 11. \end{aligned}$$

Determine the general solution set to the linear system and describe this set geometrically. [2]

3. Determine the quadratic polynomial $p(t) = a_0 + a_1 t + a_2 t^2$, which interpolates the data (1,12), (2,15), (3,16). That is, determine a_0, a_1, a_2 such that the following equations:

$$a_0 + a_1(1) + a_2(1)^2 = 12 \quad (1)$$
$$a_0 + a_1(2) + a_2(2)^2 = 15 \quad (2)$$
$$a_0 + a_1(3) + a_2(3)^2 = 16 \quad (3)$$

are satisfied.

4. Given the following augmented matrix

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 3 & h & k \end{array}\right].$$

Determine h and k such that the corresponding linear system: [3]

(a) Is inconsistent.
(b) Is consistent with infinitely many solutions.
(c) Is consistent with a unique solution.

5. Given the matrix $\mathbf{A}$ and the vector $\mathbf{b}$.

$$\mathbf{A} = \begin{bmatrix} 5 & 3 \\ -4 & 7 \\ 9 & -2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 22 \\ 20 \\ 15 \end{bmatrix}$$

Are there constants x_1 and x_2 such that $\mathbf{b}$ can be formed as a linear combination of the columns of $\mathbf{A}$? If so then what are they?[4]

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[1]What we are trying to do here is find conditions on the coefficients a, b, c, d that will guarantee a single solution to the system. Remember that in 1-D we require that to have a unique solution to, $ax = 0$, a must be different from zero.

[2]Another way to ask this: 'Are there a set of points in $\mathbb{R}^3$ where the three previous planes intersect one another? If so, then what geometric object do the collection of these points form?' I hope that it is clear that if there a solution then these points could only form a point, line, or plane, depending on the number of free-variables you find by row-reduction.

[3]Hint: You will not need to find the reduced row-echelon form. Only the row-echelon form is needed.

[4]Another way of asking this: 'Is $\mathbf{b}$ a *linear combination* of the columns of $\mathbf{A}$?'