Homework 7 due 19 Oct. 2007
PH462 EM Waves and Optical Physics posted: 13 Oct. 2007

Reading: Heald and Marion (HM) chapter 8 and posted notes.

1) Griffiths problem 10.9. See following pages; read through example 10.2. First find the vector potential, then get the fields. You may find it helpful to use Mathematica to help with the integration.

2) HM problem 8-6

3) HM problem 8-7. Important note: there is a typo in the figure, and the angle q is supposed to be the angle between segments QP and PR.

4) HM problem 8-8. Mathematica will help. For the second part, let $\alpha = 1- \beta$, and treat that as a small expansion parameter.

5) HM problem 8-10. The result is equation 8.97. You may use mathematica to perform the integration.

6) HM problem 8-12.

Example 10.2

An infinite straight wire carries the current

$$I(t) = \begin{cases} 0, & \text{for } t \le 0, \\ I_0, & \text{for } t > 0. \end{cases}$$

That is, a constant current I_0 is turned on abruptly at $t = 0$. Find the resulting electric and magnetic fields.

Solution: The wire is presumably electrically neutral, so the scalar potential is zero. Let the wire lie along the z axis (Fig. 10.4); the retarded vector potential at point P is

$$\mathbf{A}(s, t) = \frac{\mu_0}{4\pi}\,\hat{\mathbf{z}} \int_{-\infty}^{\infty} \frac{I(t_r)}{\mathscr{r}}\, dz.$$

For $t < s/c$, the "news" has not yet reached P, and the potential is zero. For $t > s/c$, only the segment

$$|z| \le \sqrt{(ct)^2 - s^2} \tag{10.25}$$

contributes (outside this range t_r is negative, so $I(t_r) = 0$); thus

$$\begin{aligned} \mathbf{A}(s, t) &= \left(\frac{\mu_0 I_0}{4\pi}\,\hat{\mathbf{z}}\right) 2 \int_0^{\sqrt{(ct)^2 - s^2}} \frac{dz}{\sqrt{s^2 + z^2}} \\ &= \frac{\mu_0 I_0}{2\pi}\,\hat{\mathbf{z}} \ln\left(\sqrt{s^2 + z^2} + z\right)\Big|_0^{\sqrt{(ct)^2 - s^2}} = \frac{\mu_0 I_0}{2\pi} \ln\left(\frac{ct + \sqrt{(ct)^2 - s^2}}{s}\right)\hat{\mathbf{z}}. \end{aligned}$$

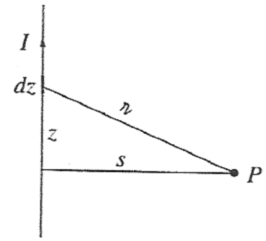

Figure 10.4

The electric field is

$$\mathbf{E}(s,t) = -\frac{\partial \mathbf{A}}{\partial t} = -\frac{\mu_0 I_0 c}{2\pi\sqrt{(ct)^2 - s^2}}\,\hat{\mathbf{z}},$$

and the magnetic field is

$$\mathbf{B}(s,t) = \nabla \times \mathbf{A} = -\frac{\partial A_z}{\partial s}\,\hat{\boldsymbol{\phi}} = \frac{\mu_0 I_0}{2\pi s}\frac{ct}{\sqrt{(ct)^2 - s^2}}\,\hat{\boldsymbol{\phi}}.$$

Notice that as $t \to \infty$ we recover the static case: $\mathbf{E} = 0$, $\mathbf{B} = (\mu_0 I_0 / 2\pi s)\,\hat{\boldsymbol{\phi}}$.

! **Problem 10.8** Confirm that the retarded potentials satisfy the Lorentz gauge condition. [*Hint:* First show that

$$\nabla \cdot \left(\frac{\mathbf{J}}{\mathscr{r}}\right) = \frac{1}{\mathscr{r}}(\nabla \cdot \mathbf{J}) + \frac{1}{\mathscr{r}}(\nabla' \cdot \mathbf{J}) - \nabla' \cdot \left(\frac{\mathbf{J}}{\mathscr{r}}\right),$$

where ∇ denotes derivatives with respect to $\mathbf{r}$, and ∇' denotes derivatives with respect to $\mathbf{r}'$. Next, noting that $\mathbf{J}(\mathbf{r}', t - \mathscr{r}/c)$ depends on $\mathbf{r}'$ both explicitly and through $\mathscr{r}$, whereas it depends on $\mathbf{r}$ only through $\mathscr{r}$, confirm that

$$\nabla \cdot \mathbf{J} = -\frac{1}{c}\dot{\mathbf{J}} \cdot (\nabla \mathscr{r}), \quad \nabla' \cdot \mathbf{J} = -\dot{\rho} - \frac{1}{c}\dot{\mathbf{J}} \cdot (\nabla' \mathscr{r}).$$

Use this to calculate the divergence of $\mathbf{A}$ (Eq. 10.19).]

! **Problem 10.9**

(a) Suppose the wire in Ex. 10.2 carries a linearly increasing current

$$I(t) = kt,$$

for $t > 0$. Find the electric and magnetic fields generated.

(b) Do the same for the case of a sudden burst of current:

$$I(t) = q_0 \delta(t).$$