2_27_08

Note Title 2/26/2008

Let $V(x) = -\operatorname{sech}^2(x)$

Let $\hbar = 1$ $m = \frac{1}{2}$ TISE $\Rightarrow$

$$-\frac{d^2}{dx^2}\psi(x) - 2\operatorname{sech}^2(x)\psi(x) = E\psi(x)$$

1) Show that

$$\psi(x) = e^{ikx}\left(\operatorname{Tanh}(x) + C\right)$$

is a solution for some C.

Solution: recall

$$\frac{d}{dx}\tanh(x) = \operatorname{sech}^2(x)$$

So $$\frac{d^2}{dx^2}\psi(x) = -e^{ikx}\left[k^2\left(C + \operatorname{Tanh}(x)\right) + 2\operatorname{Sech}^2(x)\left(\operatorname{Tanh}(x) - ik\right)\right]$$

$$\Rightarrow -\psi'' - 2\operatorname{sech}^2(x)\psi(x) =$$

🚩 $$e^{ikx}\left[k^2\left(C + \operatorname{Tanh}(x)\right) + 2\operatorname{Sech}^2(x)\left[\operatorname{Tanh}(x) - ik\right]\right]$$

$$-2\operatorname{sech}^2(x)\left(\operatorname{Tanh}(x) + C\right)e^{ikx} = Ee^{ikx}\left(\operatorname{Tanh}(x) + C\right)$$

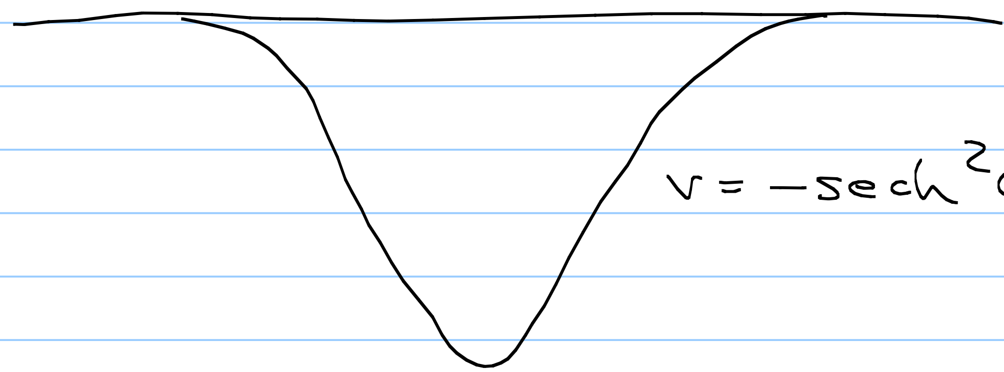
So if $C = -ik$ then 🚩 reduces to

$$k^2 e^{ikx}\left(\operatorname{Tanh}(x) - ik\right) = Ee^{ikx}\left(\operatorname{Tanh}(x) - ik\right)$$

$$\Rightarrow \qquad E = k^2$$

So $\psi(x) = e^{ikx}\left(\operatorname{Tanh}(x) - ik\right)$

$E = k^2 > 0$

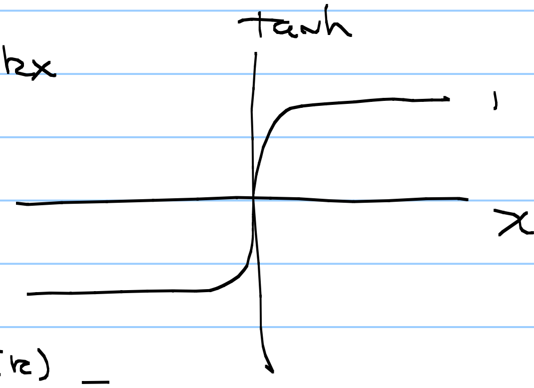


$V = -\operatorname{sech}^2(x) < 0$

Now compute the S matrix

$$\lim_{x \to +\infty} \psi(x) = (1 - ik)\, e^{ikx}$$

$$\lim_{x \to -\infty} \psi(x) = -(1 + ik)\, e^{ikx}$$



$A = -(1 + ik)$

$F = (1 - ik)$

$$\frac{F}{A} = -\frac{(1+ik)}{1-ik} =$$

For scattering from left, we take $G = 0$:

$$R_e = \frac{|B|^2}{|A|^2} = |S_{11}|^2 = 0$$

$$T_e = \frac{|F|^2}{|A|^2} = |S_{21}| = 1$$

Symmetry

$$\begin{bmatrix} B \\ F \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} A \\ G \end{bmatrix}$$

$F = S_{21} A + G S_{22}$

$B = S_{11} A + G S_{12}$

$\xrightarrow{A}$ $\quad$ $F \rightarrow$

$\xleftarrow{} B$ $\quad$ $G \leftarrow$

$$\begin{bmatrix} B \\ F \end{bmatrix} = \begin{bmatrix} 0 & -(1-ik)/(1+ik) \\ -(1-ik)/(1+ik) & 0 \end{bmatrix} \begin{bmatrix} A \\ G \end{bmatrix}$$