Pulse broadening from dispersion

chirp = time-ordering of frequency components

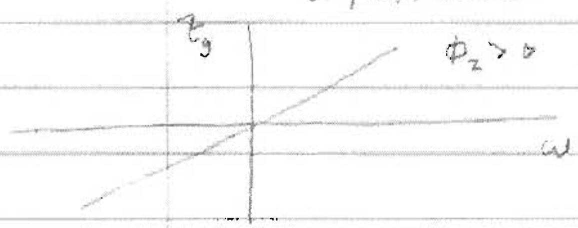
<u>spectral domain:</u>

$$\phi(\omega) = \phi_0 + \phi_1 \Delta\omega + \phi_2 (\Delta\omega)^2 + \dots \qquad w/\ \phi_n = \frac{1}{n!}\frac{\partial^n \phi}{\partial \omega^n}\Big|_{\omega}$$

$$\frac{d\phi}{d\omega} = \tau_g(\omega) = \text{group delay}$$

$$= \phi_1 + 2\phi_2 \Delta\omega + \dots$$

↳ arrival time of pulse center → freq. depend't arrival time → linear chirp



$\phi_2 > 0$ positive chirp

$\tau_g < 0$ = leading edge of pulse.

<u>time domain</u>

$$A(t) \propto \mathcal{F}^{-1}\left\{ e^{-\frac{(\omega-\omega_0)^2 \tau_0^2}{4}} e^{i(\omega-\omega_0)^2 \phi_2} \right\}$$

neglecting overall group delay (time coord moves w/ pulse)

- group coeff of $(\omega-\omega_0)^2$

$$\exp\Big(-(\omega-\omega_0)^2 \underbrace{\left(\frac{\tau_0^2}{4} - i\phi_2\right)}_{\frac{\tau_0^2}{4}(1-i\Gamma)}\Big) \qquad \Gamma = 4\phi_2/\tau_0^2$$

- inv. transform this gaussian.

$$\rightarrow \tau_p = \tau_0\sqrt{1+\Gamma^2} \qquad \text{duration is longer}$$

for propagation through material

$$\phi_2 = \frac{1}{2}\frac{\partial^2}{\partial\omega^2}(\omega n(\omega))\Big|_{\omega_0} \frac{z}{c} \qquad \propto z.$$

$\therefore$ $\tau_p(z)$ just like Gaussian beam $w(z)$

intensity $\propto 1/\tau_b(z)$

temporal phase $\phi(t) = -bt^2$

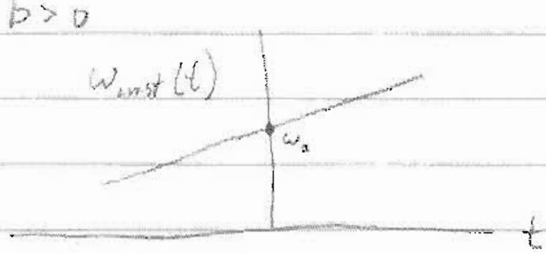
$b = \Gamma/\tau_b^2$

note that total phase is $-\omega_0 t - bt^2$

instantaneous frequency:

$$\omega_{inst}(t) = -\frac{\partial\phi}{\partial t} = \omega_0 + 2bt$$

for $b>0$



$\omega_{inst} < \omega_0$ for $t<0$

red first = pos. chirp.

at $t = \tau_b$

$$\omega_{inst} - \omega_0 = 2\frac{\Gamma}{\tau_b^2}\tau_b$$

for strongly chirped pulse,

$$\tau_b = \tau_0\sqrt{1+\Gamma^2} \approx \tau_0\Gamma$$

$$\omega_{inst} - \omega_0 = \frac{2\Gamma}{\tau_0\Gamma} = \frac{2}{\tau_0} = \delta\omega$$

illustrates $t-\omega$ correspondance for chirped pulses.

So what is the impulse response? For a range of input freq:

$$P^{(1)}(\omega) = \chi^{(1)}(\omega)\, E(\omega)$$

Alternative method: take FT of 2nd order eqn.

note that $\mathfrak{F}\{\partial f/\partial t\} = -i\omega F(\omega)$

$\rightarrow$ eqn for $\chi^{(1)}(\omega)$ with $E(\omega)$ driving

Now in time domain,

$$P^{(1)}(t) = \mathfrak{F}^{-1}\{\chi^{(1)}(\omega)\, E(t)\}$$

Recognize $\chi^{(1)}(\omega)$ as a transfer function.

let $R^{(1)}(t) \equiv 2\pi\,\mathfrak{F}^{-1}\{\chi^{(1)}(\omega)\} \equiv$ impulse response

Then $$P^{(1)}(t) = \epsilon_0 \int_{-\infty}^{\infty} R^{(1)}(\tau)\, E(t-\tau)\, d\tau = R^{(1)} \otimes E$$

(normally have $\frac{1}{2\pi}$ in front, this is absorbed into def'n of R)

What is the nature of $R^{(1)}(t)$?

- causality requires $R^{(1)}(t) = 0$ for $t < 0$
- expect? damped SHO response to a kick:

- Proof: 1) $\mathfrak{F}^{-1}\{\chi^{(1)}(\omega)\} = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} \frac{N(e^2/m)}{\omega_0^2 - \omega^2 - 2i\omega\gamma}\, e^{-i\omega t}\, d\omega$

requires contour integration: poles are off real axis:

for $t < 0$ close on upper half $\Rightarrow 0$

for $t > 0$ close lower.

Time-dependent NL response

generalize 1st order expression:

$$P^{(2)}(t) = \int_0^\infty d\tau_1 \int_0^\infty d\tau_2\, R^{(2)}(\tau_1, \tau_2)\, E(t-\tau_1)\, E(t-\tau_2)$$

causality: $R^{(2)} = 0$ for either τ_1 or $\tau_2 < 0$

put $E \to \omega$ domain:

$$P^{(2)}(t) = \int_{-\infty}^{\infty} \frac{d\omega_1}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_2}{2\pi} \int_0^\infty d\tau_1 \int_0^\infty d\tau_2\, R^{(2)}(\tau_1, \tau_2)\, E(\omega_1)\, e^{-i\omega_1(t-\tau_1)}\, E(\omega_2)\, e^{-i\omega_2(t-\tau_2)}$$

define $$\chi^{(2)}(\omega_3; \omega_1, \omega_2) = \int_0^\infty d\tau_1 \int_0^\infty d\tau_2\, R^{(2)}(\tau_1, \tau_2)\, e^{i(\omega_1\tau_1 + \omega_2\tau_2)}$$

$$= \tilde{\mathcal{F}}_{12}\left\{ R^{(2)}(\tau_1, \tau_2) \right\}$$ 2D transform

$R^{(2)}$ truncated $\tau_1, \tau_2 < 0$

$$\therefore P^{(2)}(t) = \tilde{\mathcal{F}}_{12}^{-1}\left\{ \chi^{(2)}(\omega_3; \omega_1, \omega_2)\, E(\omega_1)\, E(\omega_2) \right\}$$

if response is instantaneous $\chi^{(2)}$ is dispersionless.

$R^{(2)}(\tau_1, \tau_2) = \chi^{(2)}\, \delta(\tau_1)\, \delta(\tau_2)$

$P^{(2)}(t) = \chi^{(2)} E_1(t)\, E_2(t)$

Many systems have delayed response.

Doubling and mixing with wide bandwidth pulses.

1) short pulse, no dispersion or Δk

$$E_2(t) \propto (E_1(t))^2 \qquad e^{-2t^2/\tau_p^2}$$

for gaussian:

harmonic pulse is $\sqrt{2}$ shorter.

bandwidth?

$\tau_p \, \delta\omega = 2$ for Gaussian.

$\therefore$ if $\tau_{p2} = \frac{1}{\sqrt{2}} \tau_{p1}$ $\quad \delta\omega_2 = \sqrt{2}\, \delta\omega_1$

we measure $\delta\lambda$:

$$\frac{\delta\lambda}{\lambda} = \frac{\delta\omega}{\omega} \rightarrow \delta\lambda = \frac{\lambda^2}{2\pi c} \delta\omega = \frac{\lambda^2}{2\pi c} \frac{2}{\tau_p}$$

$$\delta\lambda_2 = \frac{\lambda_2^2}{\pi c} \frac{1}{\tau_{p2}} = \left(\frac{\lambda_1}{2}\right)^2 \frac{1}{\pi c} \frac{\sqrt{2}}{\tau_{p1}} = \frac{\lambda_1^2 \sqrt{2}}{4} \frac{1}{\pi c} \frac{\pi c \, \delta\lambda_1}{\lambda_1^2}$$

$$\therefore \delta\lambda_2 = \frac{\delta\lambda_1}{2\sqrt{2}}$$

2) chirped pulse, no disperson or Δk

if temporal profile ~ spectrum,

expect $\delta\omega_2 = \delta\omega_1/\sqrt{2}$

and minimum, compressed duration is $\tau_{p2} = \sqrt{2}\, \tau_{p1}$

$\therefore$ pulse is shortest by maximizing $\delta\lambda_2$

time domain:

$$A(t) \propto \exp\left(-t^2/\tau_b^2 + i b t^2\right)$$

τ_b = chirped pulse duration