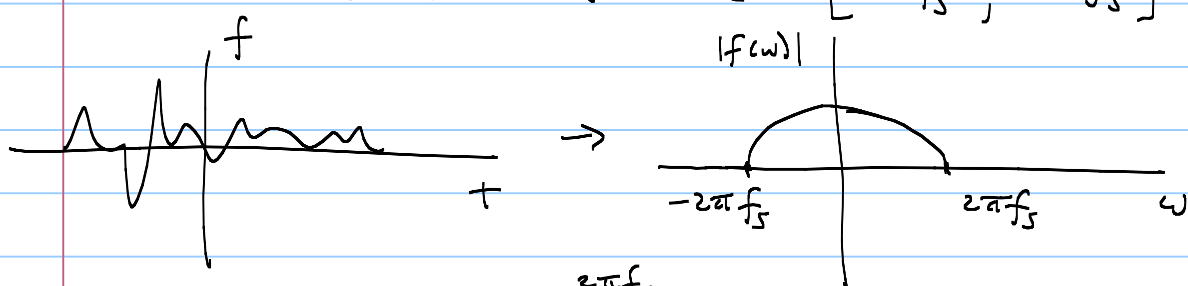
11-2-07

Note Title 11/2/2007

Band-Limited functions

have finite frequency information
i.e. if $f(t)$ is Band limited then $F(\omega)$ is exactly zero outside of some interval.

Call this interval $[-2\pi f_s, 2\pi f_s]$



So $$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-2\pi f_s}^{2\pi f_s} F(\omega)\, e^{i\omega t}\, d\omega$$

This means that $F(\omega)$ itself can be written as a Fourier Series

$$F(\omega) = \sum_{n=-\infty}^{\infty} \phi_n e^{i\omega n/2f_s}$$

where $$\phi_n = \frac{1}{2\cdot 2f_s} \int_{-2\pi f_s}^{2\pi f_s} F(\omega)\, e^{-i\omega n/2f_s}$$

compare with

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-2\pi f_s}^{2\pi f_s} F(\omega)\, e^{i\omega t}\, d\omega$$

This says that

$$4 f_s \phi_N = \sqrt{2\pi}\, f\left(\tfrac{N}{2f_s}\right)$$

$$\phi_N = \frac{\sqrt{2\pi}}{4} \frac{f(N/2f_s)}{f_s}$$

<u>Recall</u>

$$F(\omega) = \sum^{\infty} \phi_N e^{i\omega N/2f_s}$$

$$F(\omega) = \frac{\sqrt{2\pi}}{4} \sum_{N=-\infty}^{\infty} \frac{f(N/2f_s)}{f_s} e^{i\omega N/2f_s}$$

$$\Rightarrow f(t) = \frac{1}{4} \sum_{N=-\infty}^{\infty} \frac{f(N/2f_s)}{f_s} \int_{-\infty}^{\infty} e^{i\omega[N/2f_s - t]} dt$$

skipping steps

$$\sum_{N=-\infty}^{\infty} f(N/2f_s) \frac{\sin(\pi(2f_s t - N))}{\pi(2f_s t - N)}$$

sinc function interpolation
See Mathematica N.B.