Reading Today: 9.3

Tomorrow: 9.4

EM Waves in matter

If there's no free charge or free current (non-conducting material)

$$\vec{\nabla}\cdot\vec{D}=\emptyset \qquad \vec{\nabla}\times\vec{E}=-\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B}=\emptyset \qquad \vec{\nabla}\times\vec{H}=\frac{\partial \vec{D}}{\partial t} \rightarrow \vec{\nabla}\times\left(\frac{1}{\mu}\vec{B}\right)=\frac{\partial}{\partial t}\left(\epsilon\vec{E}\right)$$

Assuming linear materials:

$$\vec{D}=\epsilon\vec{E}=\epsilon_r\epsilon_0\vec{E} \; ; \; \vec{H}=\frac{1}{\mu}\vec{B}=\frac{1}{\mu_r\mu_0}\vec{B}$$

Eg. glass $\epsilon_r=2.25$

Eg. water $\epsilon_r=(1.33)^2$

Define index of refraction:

$$n\equiv\sqrt{\mu_r\epsilon_r}$$

You might guess is $\epsilon_0\rightarrow\epsilon$, $\mu_0\rightarrow\mu$, $c\rightarrow\frac{1}{\sqrt{\mu\epsilon}}$

$$v_{light}=\frac{c}{n}$$

Usually, $\mu_r \approx 1$ for most materials.

$$\Rightarrow n=\sqrt{\epsilon_r}$$

$$u_{em}=\frac{1}{2}\left(\epsilon E^2+\frac{1}{\mu}B^2\right) \qquad \vec{S}=\frac{1}{\mu}\vec{E}\times\vec{B}=\vec{E}\times\vec{H}$$

$$=\frac{1}{2}\left(\vec{D}\cdot\vec{E}+\vec{H}\cdot\vec{B}\right)$$

Now that we're dealing w/ matter, we can talk about reflections.

Let's start (and finish) with plane waves.

$$B=\frac{E}{v_{light}}=\frac{nE}{c} \; ; \; \frac{\omega}{k}=\frac{c}{n}$$

Boundary conditions

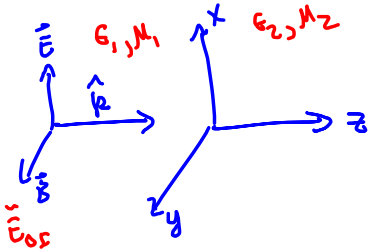
$$\epsilon_1 E_{1,\perp} = \epsilon_2 E_{2,\perp} \qquad \left(\epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}\right)$$

$$B_{1,\perp} = B_{2,\perp} \qquad \left(B_1^{\perp} = B_2^{\perp}\right)$$

$$\vec{E}_1^{\parallel} = \vec{E}_2^{\parallel}$$

$$\frac{1}{\mu_1}\vec{B}_1^{\parallel} = \frac{1}{\mu_2}\vec{B}_2^{\parallel}$$

Let's look at reflection/transmission of a normal incident wave on a dielectric interface.



Incident field

$$\vec{E}_I = \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{x}$$

$$\vec{B}_I = \tilde{B}_{0I} e^{i(k_1 z - \omega t)} \hat{y} \qquad \tilde{B}_{0I} = \frac{n_1}{c} \tilde{E}_{0I}$$

Transmitted fields

$$\vec{E}_T = \tilde{E}_{0T} e^{i(k_2 z - \omega t)} \hat{x}$$

$$\vec{B}_T = \tilde{B}_{0T} e^{i(k_2 z - \omega t)} \hat{y}$$

$$\tilde{B}_{0T} = \frac{n_2}{c} \tilde{E}_{0T}$$

Reflected fields

$$\vec{E}_R = \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} \hat{x}$$

$$\vec{B}_R = \tilde{B}_{0R} e^{i(-k_1 z - \omega t)} \hat{y}$$

$$\tilde{B}_{0R} = -\frac{n_1}{c} \tilde{E}_{0R}$$

[Solve for $\tilde{E}_{0T}$, $\tilde{E}_{0R}$ in terms of $\tilde{E}_{0I}$ and

$\beta = \frac{\mu_1 v_1}{\mu_2 v_2}$ $\quad v_1 = \frac{c}{n_1}$; $v_2 = \frac{c}{n_2}$ $\quad n_1 = \sqrt{\epsilon_{r_1} \mu_{r_1}}$

Answer: $\tilde{E}_{0R} = \left(\frac{1-\beta}{1+\beta}\right)\tilde{E}_{0I}$; $\tilde{E}_{0T} = \left(\frac{2}{1+\beta}\right)\tilde{E}_{0I}$

Intensity: $\langle \frac{1}{\mu}\vec{E}\times\vec{B}\rangle = \frac{1}{2\mu}E_0 B_0 = \frac{1}{2\mu}E_0^2 \cdot \frac{1}{v} = \frac{\epsilon}{2\mu\epsilon v}E_0^2$

$= \frac{1}{2}\epsilon v E_0^2$

$v = \frac{1}{\sqrt{\epsilon\mu}}$ Duh!

Reflection / Transmission coeff.'s.

$$R = \frac{I_{\text{reflected}}}{I_{\text{inc}}} \qquad T = \frac{I_{\text{transmitted}}}{I_{\text{inc}}}$$

$$I_{\text{ref}} = \frac{1}{2}\epsilon_1 v_1 |\tilde{E}_{0R}|^2 = \frac{1}{2}\epsilon_1 v_1 \left(\frac{1-\beta}{1+\beta}\right)^2 |\tilde{E}_{0I}|^2$$

$$\Rightarrow R = \left(\frac{1-\beta}{1+\beta}\right)^2$$

$$I_{\text{trans}} = \frac{1}{2}\epsilon_2 v_2 |\tilde{E}_{0T}|^2 = \frac{1}{2}\epsilon_2 v_2 \left(\frac{4}{(1+\beta)^2}\right)|\tilde{E}_{0I}|^2$$

$$T = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\left(\frac{4}{(1+\beta)^2}\right) = \frac{\epsilon_2 \mu_1 \mu_2 v_2}{\epsilon_1 \mu_2 \mu_1 v_1}\left(\frac{4}{(1+\beta)^2}\right)$$

$$T = \frac{\epsilon_2 \mu_1}{\epsilon_1 \mu_2}\frac{1}{\beta}\left(\frac{4}{(1+\beta)^2}\right)$$

$$= \frac{v_1^2 \mu_1 v_2}{v_2^2 \mu_2 v_1}\left(\frac{4}{(1+\beta)^2}\right) = \frac{v_1 \mu_1}{v_2 \mu_2}\left(\frac{4}{(1+\beta)^2}\right)$$

$v = \frac{1}{\sqrt{\mu\epsilon}}$ Duh again!!!

$$\boxed{= \frac{4\beta}{(1+\beta)^2}}$$ Finally!

By the by, the book was NOT wrong (big surprise) anywhere that I found

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ANSWERS!!!!

For any angle of incidence

① $\vec{k}_I, \vec{k}_R, \vec{k}_T$ and the normal to the surface all lie in the same plane.

② $\theta_I = \theta_R$

③ $\frac{1}{v_1}\sin\theta_I = \frac{1}{v_2}\sin\theta_T \quad \{n_1 \sin\theta_1 = n_2 \sin\theta_2\}$

Electric field parallel to the surface we call that transverse electric (TE).

Magnetic " " " " "

" " " " magnetic (TM).

Define: $\beta = \frac{\mu_1 n_2}{\mu_2 n_1} = \frac{\mu_1 v_1}{\mu_2 v_2}$; $\alpha = \frac{\cos(\theta_T)}{\cos(\theta_I)}$

TE: $\tilde{E}_{0R} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I}$; $\tilde{E}_{0T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I}$

$$R = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2 \quad ; \quad T = \alpha\beta\left(\frac{2}{\alpha+\beta}\right)^2$$

TM: $\tilde{E}_{0R} = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)\tilde{E}_{0I}$; $\tilde{E}_{0T} = \left(\frac{2}{1+\alpha\beta}\right)\tilde{E}_{0I}$

$$R = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2 \qquad T = \alpha\beta\left(\frac{2}{1+\alpha\beta}\right)^2$$