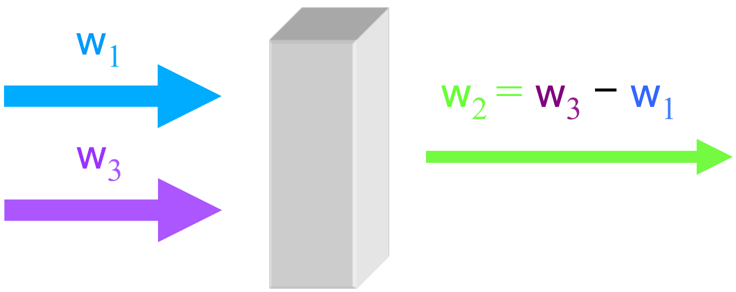
<u>7</u>
NL mixing solutions

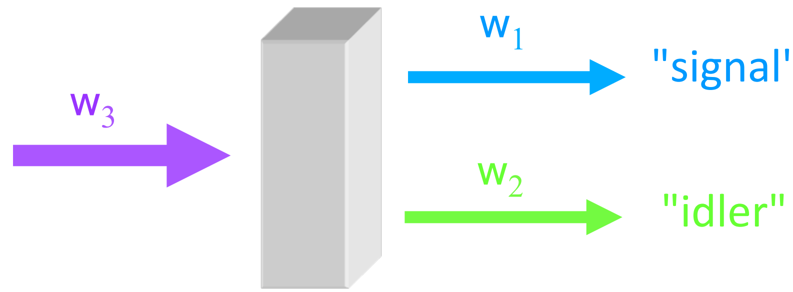
SFG, DFG, SHG, OPA, OR…
Reading for this section:
2.6-2.8
Next time:
2.10

Difference-Frequency Generation: Optical Parametric Generation, Amplification, Oscillation

Difference-frequency generation takes many useful forms.

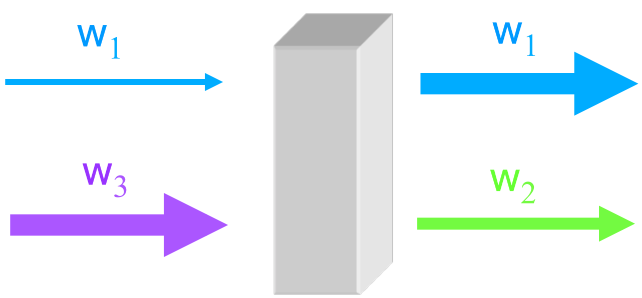


Parametric Down-Conversion
(Difference-frequency generation)

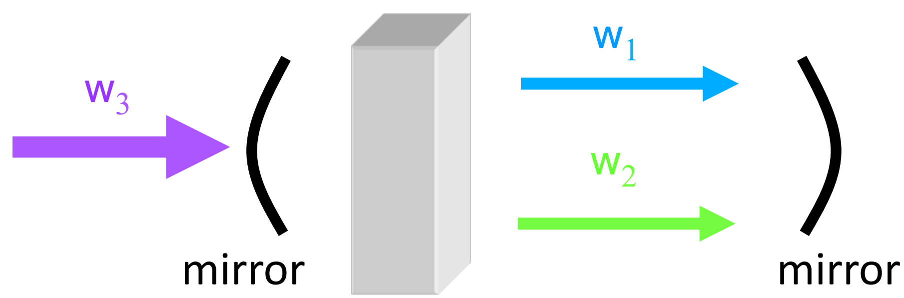


Optical Parametric Generation (OPG),
spontaneous down conversion

By convention:
$w_{signal} > w_{idler}$



Optical Parametric
Amplification (OPA)



Optical Parametric
Oscillation (OPO)

NL coupled equations for 2nd order mixing

- NL equations valid for
  - Sum frequency mixing
  - Difference frequency mixing (OPA, OPO, SPDC)
  - Define by choosing initial conditions, assumptions about which waves are the strongest

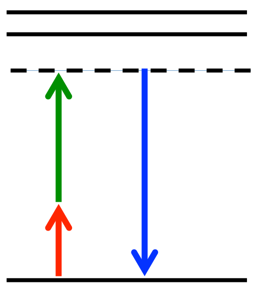
$$\frac{dA_1}{dz} = \frac{2i\, d_{eff}\, \omega_1^2}{k_1 c^2} A_3 A_2^* e^{-i\Delta k z}$$

$$\frac{dA_2}{dz} = \frac{2i\, d_{eff}\, \omega_2^2}{k_2 c^2} A_3 A_1^* e^{-i\Delta k z}$$

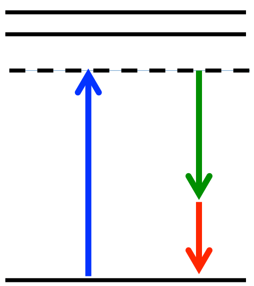
$$\frac{dA_3}{dz} = \frac{2i\, d_{eff}\, \omega_3^2}{k_3 c^2} A_1 A_2 e^{+i\Delta k z}$$

$$\Delta k = k_1 + k_2 - k_3 \qquad \omega_3 = \omega_1 + \omega_2$$

Sum frequency mixing



Difference frequency mixing



Power conservation

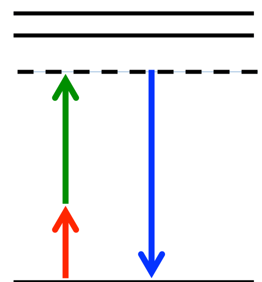
- Manley-Rowe relations (2.5)

$$\frac{d}{dz}\left(\frac{I_1}{\omega_1}\right)=\frac{d}{dz}\left(\frac{I_2}{\omega_2}\right)=-\frac{d}{dz}\left(\frac{I_3}{\omega_3}\right)$$

- Can use these to reduce the number of coupled equations
- Helpful to understand saturated conversion limits in parametric processes

Sum-Frequency mixing, non-depleted pump

- Set up mixing equations



$$\omega_3 = \omega_1 + \omega_2$$

$$\Delta k = k_1 + k_2 - k_3$$

$$\frac{dA_1}{dz} = \frac{2i\,d_{eff}\omega_1^2}{k_1 c^2} A_3 A_2^* e^{-i\Delta k z} = i\xi \frac{\omega_1}{n_1} A_3 A_2^* e^{-i\Delta k z} \qquad \xi = \frac{2 d_{eff}}{c}$$

$$\frac{dA_2}{dz} = \frac{2i\,d_{eff}\omega_2^2}{k_2 c^2} A_3 A_1^* e^{-i\Delta k z} = i\xi \frac{\omega_2}{n_2} A_3 A_1^* e^{-i\Delta k z}$$

$$\frac{dA_3}{dz} = \frac{2i\,d_{eff}\omega_3^2}{k_3 c^2} A_1 A_2 e^{+i\Delta k z} = i\xi \frac{\omega_3}{n_3} A_1 A_2 e^{+i\Delta k z}$$

Phase-matched upconversion

- Up conversion by sum-freq mixing, phase-matched, nondepleted pump $\frac{dA_2}{dz} = 0$

with $\Delta k = 0$ $\quad \frac{dA_1}{dz} = i\xi \frac{\omega_1}{n_1} A_3 A_2^* e^{-i\Delta k z} = i\xi \frac{\omega_1}{n_1} A_2^* A_3 \qquad \frac{dA_3}{dz} = i\xi \frac{\omega_3}{n_3} A_1 A_2$

Combine 2 1st order equations to one 2nd order

$$\frac{d^2 A_1}{dz^2} = i\xi \frac{\omega_1}{n_1} A_2^* \frac{dA_3}{dz} = \left(i\xi \frac{\omega_1}{n_1} A_2^* \right)\left(i\xi \frac{\omega_3}{n_3} A_1 A_2 \right)$$

$$\frac{d^2 A_1}{dz^2} = -\xi^2 \frac{\omega_1}{n_1} \frac{\omega_3}{n_3} |A_2|^2 A_1 = -\kappa^2 A_1$$

Defining the growth coefficient:

$$\kappa^2 = \xi^2 \frac{\omega_1 \omega_3}{n_1 n_3} |A_2|^2 = \frac{4 d_{eff}^2 \omega_1 \omega_3}{n_1 n_3 c^2} |A_2|^2$$

κ is real and has dimensions 1/length

Solutions for up converted signal

- Growth of signal (at shortest wavelength) follows SHO equation:

$$\frac{d^2 A_1}{dz^2} = -\kappa^2 A_1$$

General form of solution for arbitrary initial conditions:

$$A_1(z) = B\cos\kappa z + C\sin\kappa z$$

$$A_1'(z) = -B\kappa\sin\kappa z + C\kappa\cos\kappa z = i\xi\frac{\omega_1}{n_1}A_2^* A_3$$

Solve for the wave that is being upconverted:

$$A_3(z) = \frac{1}{i\xi\frac{\omega_1}{n_1}A_2^*}\left(-B\kappa\sin\kappa z + C\kappa\cos\kappa z\right) = -i\frac{n_1}{\xi\omega_1 A_2^*}\left(-B\kappa\sin\kappa z + C\kappa\cos\kappa z\right)$$

Upconversion solutions

- Solution is valid for any initial condition, as long as A_2 is constant

$$A_1(z) = B\cos\kappa z + C\sin\kappa z \qquad A_3(z) = -i\frac{n_1}{\xi\omega_1 A_2^*}\left(-B\kappa\sin\kappa z + C\kappa\cos\kappa z\right)$$

- Apply initial conditions: for upconversion, no A_3 at input, weak A_1

$$A_1(z) = A_1(0)\cos\kappa z \qquad A_3(z) = i\,A_1(0)\frac{n_1\kappa}{\xi\omega_1 A_2^*}\sin\kappa z$$

$$\left|A_3(z)\right|^2 = \left|A_1(0)\right|^2\left|\frac{n_1\kappa}{\xi\omega_1 A_2^*}\right|^2\sin^2\kappa z = \left|A_1(0)\right|^2\left|\frac{n_1 c}{2d_{eff}\ \omega_1 A_2^*}\right|^2\frac{4d_{eff}^2\omega_1\omega_3}{n_1 n_3 c^2}\left|A_2\right|^2\sin^2\kappa z$$

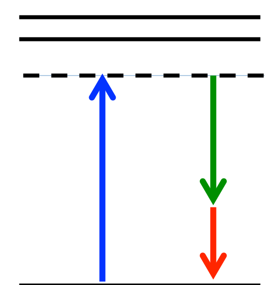
$$\left|A_3(z)\right|^2 = \left|A_1(0)\right|^2\frac{n_1\omega_3}{n_3\omega_1}\sin^2\kappa z$$

- Signal oscillates, so pick length (or pump intensity) carefully
- Max conversion corresponds to photon energy ratio

$$\left|A_3(z)\right|^2 \approx \left|A_1(0)\right|^2\frac{4d_{eff}^2\omega_3^2}{n_3^2 c^2}\left|A_2\right|^2 z^2$$ for small z

Parametric amplification

- Difference frequency mixing, no phase mismatch
- Strong, constant pump A_3



$$\omega_2 = \omega_3 - \omega_1 \quad \Delta k = k_1 + k_2 - k_3 = 0 \quad \frac{dA_3}{dz} = i\xi \frac{\omega_3}{n_3} A_1 A_2 e^{+i\Delta k z} = 0$$

$$\frac{dA_1}{dz} = i\xi \frac{\omega_1}{n_1} A_3 A_2^* e^{-i\Delta k z} \rightarrow i\xi \frac{\omega_1}{n_1} A_3 A_2^* \qquad \frac{d^2 A_1}{dz^2} = i\xi \frac{\omega_1}{n_1} A_3 \frac{dA_2^*}{dz}$$

$$\frac{dA_2}{dz} = i\xi \frac{\omega_2}{n_2} A_3 A_1^* e^{-i\Delta k z} \rightarrow i\xi \frac{\omega_2}{n_2} A_3 A_1^* \qquad \frac{dA_2^*}{dz} = -i\xi \frac{\omega_2}{n_2} A_3^* A_1$$

$$\frac{d^2 A_1}{dz^2} = \left(i\xi \frac{\omega_1}{n_1} A_3 \right)\left(-i\xi \frac{\omega_2}{n_2} A_3^* A_1 \right) = +\xi^2 \frac{\omega_1}{n_1} \frac{\omega_2}{n_2} \left| A_3 \right|^2 A_1 = +\kappa^2 A_1$$

$$\frac{d^2 A_2}{dz^2} = +\kappa^2 A_2$$

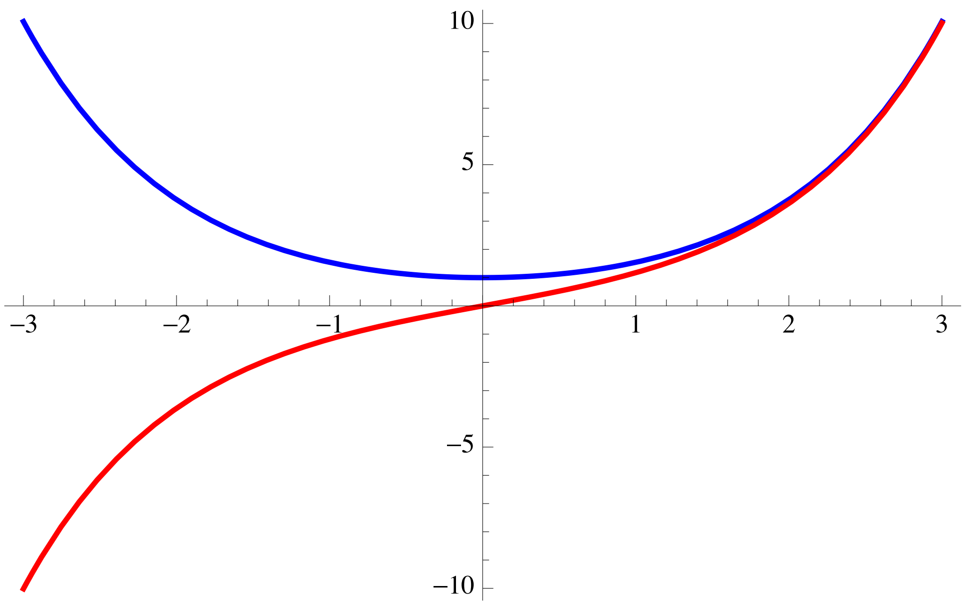
Because of positive sign, *real exponential solutions.*

Review: cosh, sinh

- Hyperbolic functions

$$\cosh(z) = \frac{1}{2}\left(e^{z} + e^{-z}\right) = \cos(iz) \qquad \sinh(z) = \frac{1}{2}\left(e^{z} - e^{-z}\right) = -i\sin(iz)$$

$$\frac{d}{dz}\cosh(az) = a\sinh(az) \qquad \frac{d}{dz}\sinh(az) = a\cosh(az)$$



Cosh(z) is even
Sinh(z) is odd

For large +z, both are exponentially increasing

Exponential gain in OPA

- Initial conditions: $A_1=A_{10}$, no input at A_2

$$A_1(z)=A_1(0)\cosh\kappa z \qquad\qquad A_2(z)=C\sinh\kappa z+D\cosh\kappa z\rightarrow C\sinh\kappa z$$

$$\frac{dA_1}{dz}=A_1(0)\kappa\sinh\kappa z=i\xi\frac{\omega_1}{n_1}A_3A_2^*=i\xi\frac{\omega_1}{n_1}A_3C^*\sinh\kappa z$$

$$C=i\frac{n_1\kappa}{\xi\omega_1A_3}A_1(0) \qquad\qquad A_2(z)=i\frac{n_1\kappa}{\xi\omega_1A_3}A_1(0)\sinh\kappa z$$

- Both signal and idler have exponential gain instead of quadratic gain.
- For κz>>1, $$I_1(z)\sim I_1(0)\exp[2\kappa z]$$
- Presence of signal stimulates production of idler and vice-versa
- Very little net conversion if process is not phase-matched: crystal phase-matching angle tunes output

Gain coefficient

- Gain can be very high:

$$2\kappa = 2\times 2d_{eff}\sqrt{\frac{\omega_1\omega_2}{c^2 n_1 n_2}}\left|A_3\right| = 4d_{eff}\sqrt{\frac{\omega_1\omega_2}{c^2 n_1 n_2}}\sqrt{\frac{I_3}{2n_3\varepsilon_0 c}}$$

$$1\text{V/m} = \sqrt{377\,\text{W/m}^2}$$

$$1W/\text{m}^2 = 10000W\ /\ cm2 = \sqrt{\frac{1}{377}}V\ /\ m$$

- For BBO

$$2\kappa \approx 4d_{eff}\frac{2\pi}{n\lambda_p/2}\sqrt{\frac{I_3}{n\varepsilon_0 c/2}} = \frac{16\pi}{n}\frac{1}{\lambda_p}d_{eff}\sqrt{\frac{I_3}{2n\varepsilon_0 c}}$$

$$d_{eff} = 2\,pm/V$$

$$n = 1.6$$

$$\kappa \approx 7\times 10^{-4}\,\frac{\sqrt{I_3\left(W/cm^2\right)}}{\lambda_p\left(\mu m\right)}\,/\,cm$$

Exp[14]=10^6
For this single pass gain and L = 5mm, need
2κ =14/L =28 /cm
Need about I_3 = 2×10^9W/cm^2