MATH 235 - Differential Equations w/ Honors January 9, 2008
Homework 1, Spring 2008 **Due**: January 14, 2008

1. Evaluate the following integrals:
   (a) $\int x^3 \cos(5x) dx$
   (b) $\int x^2 \sin(2x^3) dx$
   (c) $\int \frac{x^2}{x^2+1} dx$
   (d) $\int \frac{4-2x}{(x^2+1)(x-1)^2} dx$
   (e) $\int \frac{5x}{3x-1} dx$

   Hint: For (a)-(b) you must decide to use integration-by-parts or substitution. The fraction in (c) can be simplified by division of polynomials. The fraction in (d) can be made into a simpler sum by partial fractions. For (e) consider a substitution.

2. Assuming that $s \in \mathbb{R}$ evaluate the following improper integrals:
   (a) $\int_0^\infty x^3 e^{\beta t} e^{-sx} dt$, where $\beta \in \mathbb{R}$ and $s > \beta$.
   (b) $\int_0^\infty e^{-st} \cos(\omega t) dt$, where $\omega \in \mathbb{R}$ and $s > 0$.
   (c) $\int_0^\infty e^{-st} \sin(\omega t) dt$, where $\omega \in \mathbb{R}$ and $s > 0$.

   Hint: For (b),(c), perform two steps of integration-by-parts and watch for the original integral to reappear.

3. Solve the following equations for the variable x:
   (a) $\ln(x) - \ln(x-4) = -13$
   (b) $\ln(x) + \ln(x-4) = -13$
   (c) $e^{2(\ln(x) - \ln(x^2))} = 1$

4. Find all points (x, y), which solve the following simultaneous systems:
   (a) $$\begin{aligned} 4x - 7y - 1 &= 0 \\ 3x + 6y - 1 &= 0 \end{aligned}$$
   (b) $$\begin{aligned} 2x - \frac{2x^2}{3} - xy &= 0 \\ 4xy - 16y &= 0 \end{aligned}$$
   (c) $$\begin{aligned} yx^2 + y^3 - y &= 0 \\ x - x^3 - xy^2 &= 0 \end{aligned}$$

5. Consider the first order linear homogenous ordinary differential equation(ODE), $\frac{dy}{dt} = (1+t)y$. We can think of ODEs as equations which define the slope of the solution $y(t)$ for a given point (t, y).
   (a) Using HPGSolver plot the vector field associated with the ODE.
   (b) Using HPGSolver plot solution curves passing through the points $(t_0, y_0) = \{(-1,2), (-1,-2), (-1,0), (-\frac{1}{2}, 3), (-\frac{1}{2}, -3), (-2,2), (-2,-2), (1,1), (-1,1)\}$.
   (c) Verify that $y(t) = Ce^{\frac{1}{2}t^2 + t}$ satisfies the ODE regardless of the choice of the constant C.