Midterm Take Home 2011

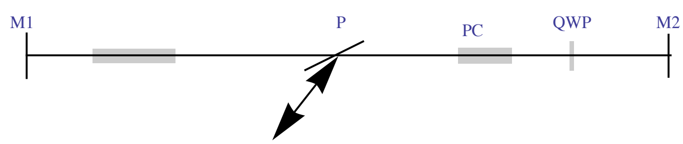
Midterm rules:
You can use your notes, the textbook (Svelto) and my notes that are online.
You can't use other resources, including the internet, other people, texts, etc.
Check the webisite frequently to see if there are any clarifications posted.
Email me or talk to me if you have questions.

- **1. Multipass amplifier design**

  In this problem, you will work through several aspects of the design of a laser amplifier for pulses. A simple model of a regenerative amplifier is shown in the figure. For the purposes of the problem, all you really need to know is that the amplifier is configured so that a laser pulse can be made to pass through the crystal a controllable number of times, so that the pulse energy can build up to the energy level that is desired.

  In this problem you will compare two gain media, Nd:YAG and Yb:YAG. YAG is the host crystal and Nd and Yb are the doping ions that are active in the gain. These parameters of these materials are given in tables 9.3 and 9.4 in Svelto. (Although Yb:YAG is a quasi- 3 level system, we will ignore any population of the lower laser level in this problem.)

  For further background: The laser crystal is placed inside of a resonator with two high-reflecting mirrors M1 and M2. Also inside the resonator is a polarizing beamsplitter, a quarter-waveplate (QWP) and a device called a Pockels cell. On a double-pass, the quarter waveplate rotates the polarization by 90 degrees. The Pockels cell is essentially a fast, voltage-controlled quarter waveplate: when it is activated, it cancels the effect of the static quarter waveplate. A pulse enters the laser cavity by reflecting off the polarizer; double passing the QWP allows it to get past the polarizer to the laser crystal. While the pulse is on that side of the resonator, the pockels cell is switched on, and cancels the QWP, trapping the pulse into the resonator. When the desired number of passes has occurred, the PC is turned back off, and the amplified pulse leaves the resonator in the same direction as it came in.



- **a. Assuming the ion densities in Tables 9.3 and 9.4 of Svelto for these materials, what crystal thickness L do we need to be able to absorb 95% of the pump? The absorption cross-sections and pump wavelengths for these materials are:**

  Nd:YAG $\sigma_a = 7.7 \times 10^{-20}\ \mathrm{cm}^2 \quad \lambda_p = 808\ \mathrm{nm}$

  Yb:YAG $\sigma_a = 2.6 \times 10^{-20}\ \mathrm{cm}^2 \quad \lambda_p = 976\ \mathrm{nm}$

- **b. Calculate the gain saturation fluence for the two materials in J / cm^2. Show your work.**

- **c. Assume that the laser crystal is pumped by a short laser pulse that instantaneously creates a population inversion density of $N_{\mathrm{inv}} = N_2 - N_1$, evenly distributed through the length of the crystal L and a circular cross-sectional area A_{pump}. The stimulated emission cross-section for the laser medium is known, σ_{21}. The input and output energies of the pulse being amplified are E_{in} and E_{out}.**

  - ***In terms of these parameters,*** **write an expression for the small-signal, single-pass gain $G = E_{\mathrm{out}} / E_{\mathrm{in}}$, assuming the beam being amplified completely overlaps that of the pump in the crystal.**

- Write an alternate expression for G in terms of the saturation fluence, and explain the meaning of the other term(s) that show up in this expression.

- **d. The pump pulse has an energy of 10mJ.**

  - What pump spot radius w_p do we need to use to get $G = 3$ for Nd:YAG?

  - If we have the same pump size and energy for Yb:YAG, what is G?

- **e. In both cases, what is the theoretical maximum energy we could extract from the amplifiers?**

- **f. Assume the initial energy of the pulse to be amplified is 1nJ and that the pump and amplified beams are the same size.**

  - Ignoring saturation, how many amplifier round trips (integer number, 2 passes per round trip) do we need to be able to just surpass half this limit?

  - Calculate the ratio of the fluences of the amplified pulse at the *input* to the last *pass* to the saturation fluence of the gain media.

- **g. Each pass of the pulse removes some stored energy from the crystal. By removing the energy extracted by each pass from the stored energy in the crystal, calculate the new gain for each pass.**

  - In this way, calculate and *plot* the pulse energy for each *pass* through the crystal for the two gain media. This will be easiest to do with a For[ ] loop and lists. You can modify the code below.

  - What is the new number of round trips that are required to extract at least half of the stored energy for both gain media?

```
nJ = 10^-9;
nPasses = 10;
EpulseLst = gainLst = 0 Range[nPasses + 1];

EpulseLst[[1]] = 1 nJ;
gainLst[[1]] = 10;
For[i = 1, i ≤ nPasses, i++,
  EpulseLst[[i + 1]] = gainLst[[i]] EpulseLst[[i]];
  gainLst[[i + 1]] = Max[ gainLst[[i]] - 1, 1]
  (* make sure it isn't < 1, put acutal modification to gain here *)
 ];
ListPlot[EpulseLst, Joined → True]
```

- **h. This calculation assumes that the pulse experiences the full small signal gain on each pass.**
  - Assess for each case whether this is a good assumption.
  - How could we extend this numerical model to include stored energy depletion within the pulse?
  - List 3 other assumptions that are implicit in this model (there are many more than 3), and describe qualitatively how the results would be affected by making the model more realistic.

- **Extra credit:**
  Extend the code to account for gain depletion within the amplified pulse, assuming the pulse shape in time is Gaussian.

- **2. Single longitudinal mode laser.**

- **a. Write an expression for the longitudinal mode spacing for a linear resonator for a solid state laser.**

  - Estimate the maximum cavity length *L* for a Nd:YAG laser (crystal length *l*) that will ensure that the laser will support only one longitudinal mode.

  - Calculate the exact cavity length that will give maximum gain for the wavelength at the gain maximum.

- **b. Assume that the stimulated emission cross section depends on the laser frequency with a Lorentzian lineshape:**
  $\sigma(\nu) = \frac{\sigma_{21}}{1+\left(\frac{(\nu-\nu_0)}{\Delta\nu_t}\right)^2}$
  Write an expression for the output power as a function of the pump power that incorporates the laser operating frequency ν.

- **c. Suppose the above laser oscillator is pumped at 6x the threshold power for the center wavelength.**
  - How far can we tune the laser output frequency while keeping the laser above threshold?
  - Make a plot of the relative output power vs laser frequency (i.e. normalize the plot so that $P_{out}(\nu_0) = 1$).

- **d. As you are tuning the laser, it is possible for another longitudinal mode to have enough gain to lase. If this happens, the laser will hop to the frequency corresponding to that mode.**
  Check to see if the cavity length you estimated in part a still insures that the cavity will always lase only on the intended mode. Find a new cavity length that prevents mode hopping within the tuning range.