Final Spring 08 Name ______________________

For credit you must explain your reasoning in all problems.

1. Derive an integral expression for the dipole moment of a disc of radius R with $\rho = \rho_0 \cos\theta$.

$\boxed{\text{Defn:}}$ $\vec{p} = \int \vec{r}\, dq = \int \vec{r} \rho\, d\tau$ Sum vector that locates dq after multiplying by dq over all charge

$\vec{r} = r\cos\theta\, \hat{x} + r\sin\theta\, \hat{y}$ $\quad d\tau = r\, d\theta\, dr$

$$\vec{p} = \int_0^R \int_0^{2\pi} (r\cos\theta\, \hat{x} + r\sin\theta\, \hat{y})\, r\, d\theta\, dr$$

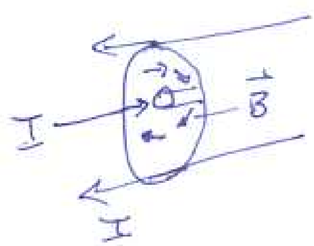
remember cartesian unit vectors come outside integral!

2. A long coaxial cable carries current I (current flows down the surface of the inner cylinder, radius a, and back along the outer cylinder, radius b). Derive an integral expression for the energy stored in length L.

Energy stored in B field since there is no E field

$u = \frac{1}{2\mu_0} B^2 \left(\frac{J}{m^3}\right)$ energy density. Total energy $= \int u\, d\tau = U$

Find B from Amps Law. $\oint \vec{B}\cdot d\vec{l} = \mu_0 I \Longrightarrow B\, 2\pi r = \mu_0 I$



$$U = \frac{1}{2\mu_0} \int_0^L \int_a^b \int_0^{2\pi} \frac{\mu_0^2 I^2}{4\pi^2 r^2}\, r\, d\phi\, dr\, dL$$

3. An iron spherical magnet is magnetized. It carries magnetization $M = M_0\hat{z}$. Explain how you would derive an expression for B everywhere?

ONLY bound current, no free current. Current determines $\vec{B}$ via Biot-Savart (or finding A then $\vec{B} = \vec{\nabla} \times \vec{A}$)

$\vec{J}_b = \vec{\nabla} \times \vec{M} = 0 \qquad \vec{K}_b = \vec{M} \times \hat{n}$