

## Dispersion

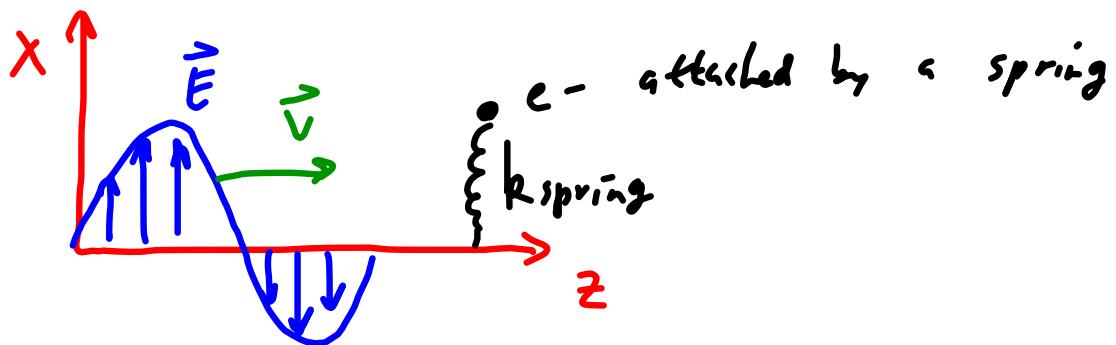
$$V_{\text{wave}} = V_{\text{phase}} = \frac{\omega}{k}$$

Speed that each sinusoidal component has.

Packet or envelope travels at the group velocity

$$v_g = \frac{d\omega}{dk} \quad \omega = v_{\text{phase}} k$$

## Simple Model of the index of Refraction



2<sup>nd</sup> Law:

$$m \frac{d^2x}{dt^2} = F_{\text{binding}} + F_{\text{damping}} + F_{\text{driving}}$$
$$= -\underbrace{m\omega_0^2 x}_{k_{\text{spring}}} - m\gamma \frac{dx}{dt} + q E_0 \cos(\omega t)$$

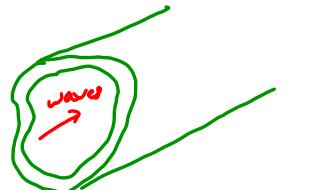
## Cauchy's Formula

$$n = 1 + A + \frac{AB}{\lambda^2}$$

### Wave Guide

Look at Conductors first

Inside:  $\vec{E} = 0, \vec{B} = 0$



B.C.  $\Rightarrow \vec{E}' = 0, \vec{B}' = 0$

generate current  
which takes away  
energy

generate  
eddy currents



Look for solutions of the form:

$$\tilde{\vec{E}} = \tilde{E}_0 e^{i(kz - \omega t)} \quad \text{and} \quad \tilde{\vec{B}} = \tilde{B}_0 e^{i(kz - \omega t)}$$

$$\tilde{\vec{E}}_0 = E_x \hat{x} + E_y \hat{y} + E_z \hat{z} \quad \tilde{\vec{B}}_0 = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$$

Put these solutions into Faraday's Law to find  
the relationships between  $\{E_x, E_y, E_z\}$   $\{B_x, B_y, B_z\}$

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\hat{z}: \frac{\partial}{\partial x} [E_y e^{i(kz - \omega t)}] - \frac{\partial}{\partial y} [E_x e^{i(kz - \omega t)}] = -B_z (-\omega) e^{i(kz - \omega t)}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = +i\omega B_z$$

$$\hat{x}: \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = i\omega B_x$$

$$\frac{\partial E_z}{\partial y} - ik E_y = i\omega B_x$$

Next

Do Ampere's Law

$\hat{y}:$

$$\begin{aligned}\partial_x E_y - \partial_y E_x &= i\omega B_z & \partial_x B_y - \partial_y B_x &= -\frac{i\omega}{c^2} E_z \\ \partial_y E_z - ik E_y &= i\omega B_x & \cancel{\partial_y B_z - ik B_y} &= -\frac{i\omega}{c^2} E_x \\ ik E_x - \partial_x E_z &= i\omega B_y & \cancel{ik B_x - \partial_x B_z} &= -\frac{i\omega}{c^2} E_y\end{aligned}$$

Solve for

$E_x \& E_y$  in terms of  $\perp$  components

$$E_x = \frac{i}{(\omega/c)^2 - k^2} [k \partial_x E_z + \omega \partial_y B_z]$$

$$E_y = \frac{i}{(\omega/c)^2 - k^2} [k \partial_y E_z - \omega \partial_x B_z]$$

Using  $\vec{\nabla} \cdot \vec{E} = 0 \& \vec{\nabla} \cdot \vec{B} = 0$

$$\text{get: } [\partial_x^2 + \partial_y^2 + (\omega/c)^2 - k^2] E_z = 0$$

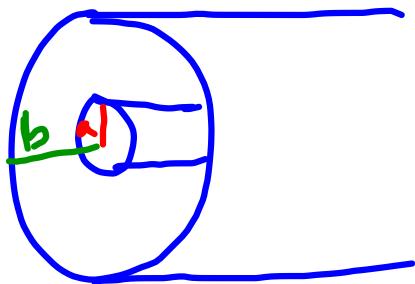
$$[\partial_x^2 + \partial_y^2 + (\omega/c)^2 - k^2] B_z = 0$$

TE = Transverse electric field  $E_z = 0$

TM = " Magnetic "  $B_z = 0$

TEM =  $E_z = 0, B_z = 0$

## Coaxial Transmission Line



TEM solution

$$B_z = 0, E_z = 0$$

$$\Rightarrow \vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \times \vec{E} = 0$$

Electrostatics

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = 0$$

Magnetostatics

$\vec{E}$  = due to an infinite line charge

$\vec{B}$  = " " " " " current

$$\vec{E} = \frac{A}{s} \cos(kz - \omega t) \hat{s}$$

$$\vec{B} = \frac{A}{sc} \cos(kz - \omega t) \hat{\varphi}$$

# Fiber optic / optical waveguide

