Note Title 2/26/2010

Review:

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(z')}{r} \sum_{l=0}^{\infty} \left(\frac{z'}{r}\right)^l P_l(\cos\theta)\, dz' = \frac{1}{4\pi\epsilon_0} \sum_{l=0}^{\infty} \frac{M_l P_l(\cos\theta)}{r^{l+1}}$$

where $M_l = \int \lambda(z')(z')^l dz'$

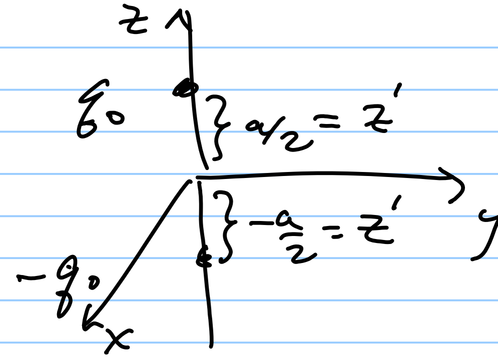
monopole moment

$$V_0 = \frac{1}{4\pi\epsilon_0} \frac{Q_{tot}}{r}$$

Dipole potential

$$V_1 = \frac{1}{4\pi\epsilon_0} \frac{M_1 P_1(\cos\theta)}{r^2}$$

Example:



$$\lambda(z') = q_0\,\delta\!\left(z' - \frac{a}{2}\right) - q_0\,\delta\!\left(z' + \frac{a}{2}\right)$$

Dipole

$$M_1 = \int q_0\,\delta\!\left(z' - \frac{a}{2}\right) z'\,dz' - \int q_0\,\delta\!\left(z' + \frac{a}{2}\right) z'\,dz'$$

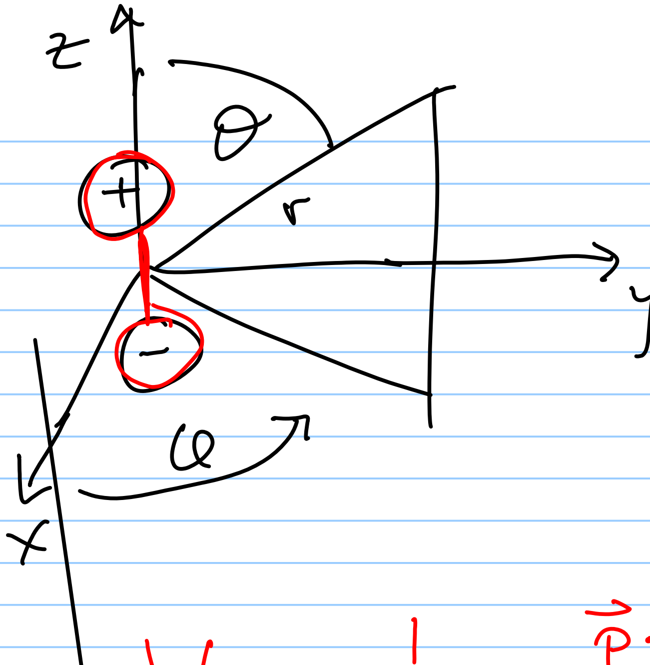
↑ blows up a $z' = a/2$ blows up at $z' = -a/2$

$$M_1 = q_0 \frac{a}{2} - q_0\left(-\frac{a}{2}\right) = q_0 a$$

$$V_1 = \frac{1}{4\pi\epsilon_0} \frac{M_1 \cos\vartheta}{r^2}$$

$P \equiv M_1$

↑ dipole moment



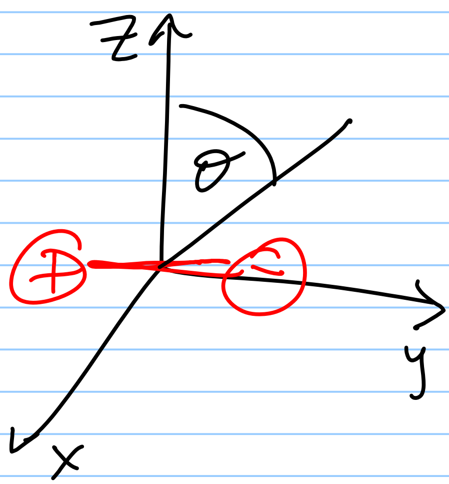
ξ_1

← define vector from ⊖ to ⊕ with magnitude M, so

$\vec{P}$ then $\vec{p}\cdot\hat{r} = |\vec{p}||\hat{r}|\cos\theta$

$$|\vec{p}| = p \qquad |\hat{r}| = 1$$

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$



What is V_1 in the y-z plane?

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{p\cos(\pi/2+90)}{r^2}$$

atomic polarizability: INDUCED $\vec{p}$ (not permanent $\vec{p}$)

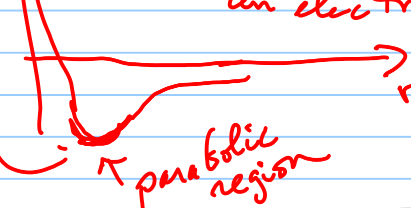
units $\quad E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{C}{m^2}$

Expt: $\quad \vec{p} \simeq \alpha \vec{E} + \beta E^2 \hat{E} + \gamma E^3 \hat{E}$

non-linear terms

from being away from

Total Energy of atom modelled as an electron in a square well of width r.



qs has units C·m

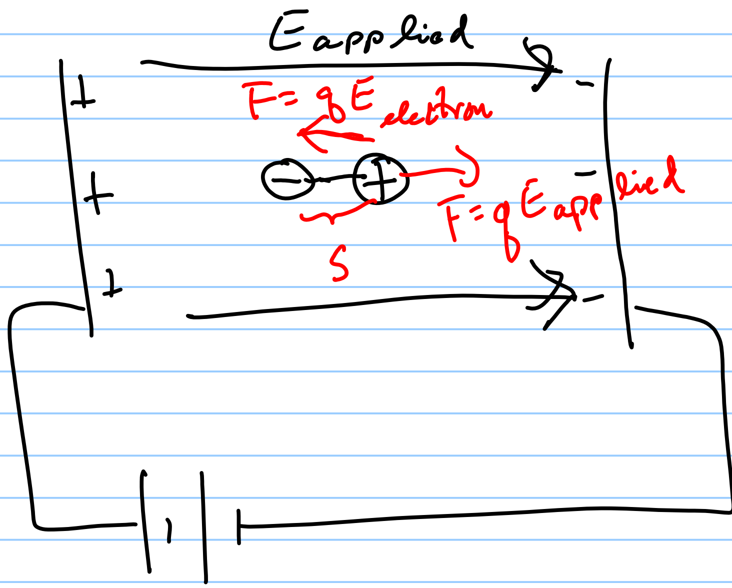
H	He	Li	Be	C	Ne	Na	Ar	K	Cs
.667	.205	24.3	5.6	1.76	.396	24.1	1.64	43.4	59.6

Atomic Polarizabilities $\frac{\alpha}{4\pi\epsilon_0}$ units $10^{-30}\ m^3$

For hydrogen

$$\frac{\alpha}{4\pi\epsilon_0} = .667 \times 10^{-30}\ m^3 \qquad \alpha = .667\ 4\pi\epsilon_0 \times 10^{-30}\ m^3$$

Simple model to predict α: Free body diagram



$$qE_{electron} = qE_{applied}$$

$$p = \alpha E_{applied}$$

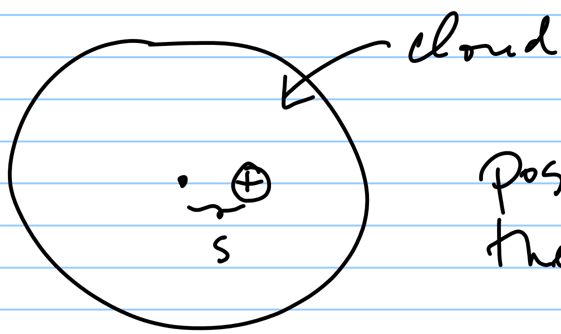
$$\shortparallel$$

$$qs = \alpha E_{electron} = \alpha \frac{1}{4\pi\epsilon_0}\frac{q}{s^2}$$

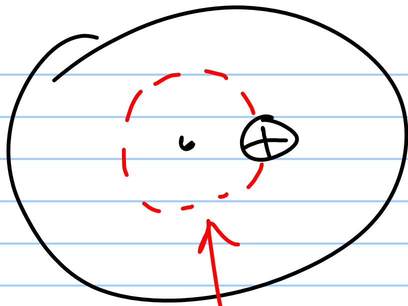
$$\alpha = 4\pi\epsilon_0 s^3 \quad \text{not a constant!}$$

More complicated model: assume electron cloud with charge uniformly distributed out to Bohr radius a:

$$\rho = \frac{q}{\frac{4}{3}\pi a^3}$$



Positive charge sees an E at a distance s away from the center of the electron charge



Gaussian spherical surface to find E

$$\Phi = Q_{enc}/\epsilon_0$$

$$\underset{\uparrow\ E_{electron}}{E}\, 4\pi s^2 = \frac{\rho\, \frac{4}{3}\pi s^3}{\epsilon_0} = \frac{\overset{\swarrow \rho}{\frac{q}{\frac{4}{3}\pi a^3}}\ \frac{4}{3}\pi s^3}{\epsilon_0} = \frac{q\, s^2}{\epsilon_0 a^3}$$

$$E_{electron} = \frac{q}{4\pi\epsilon_0}\frac{s}{a^3}$$

Static equilibrium $\Rightarrow$ $E_{applied} = E_{electron}$

$$p = \alpha E_{applied}$$

$$\shortparallel$$

$$q s = \alpha \frac{q s}{4\pi\epsilon_0 a^3} \Rightarrow \alpha = 4\pi\epsilon_0 a^3$$