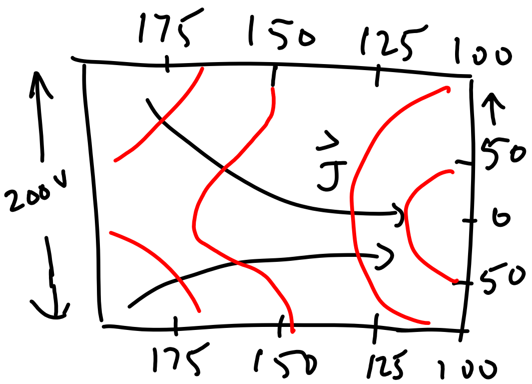
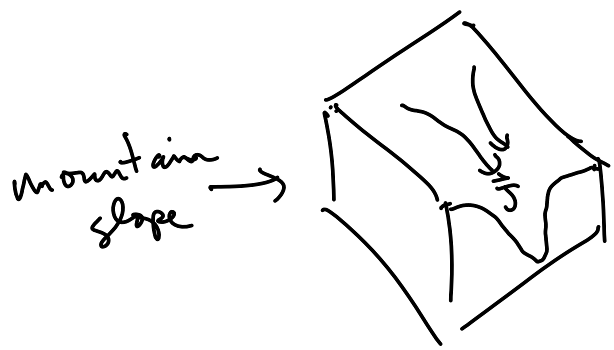
Homework 10 solutions

Homework problem 1.) Design the boundary values to focus the current in the center of your boundary as the current travels in an Ohmic material from the left to the right side. Use the relaxation method and include a printout of your spreadsheet. How would you determine how much thermal power (lecture Feb 24) is delivered at the "focal point?"

$\vec{E}$ xamples





See Feb 24 lecture to get power density $\propto \frac{J^2}{\sigma} \; \frac{\text{Joules}}{s\, m^3}$

This power is converted to heat in the material. It is distributed throughout but large where the lines of E or J are concentrated.

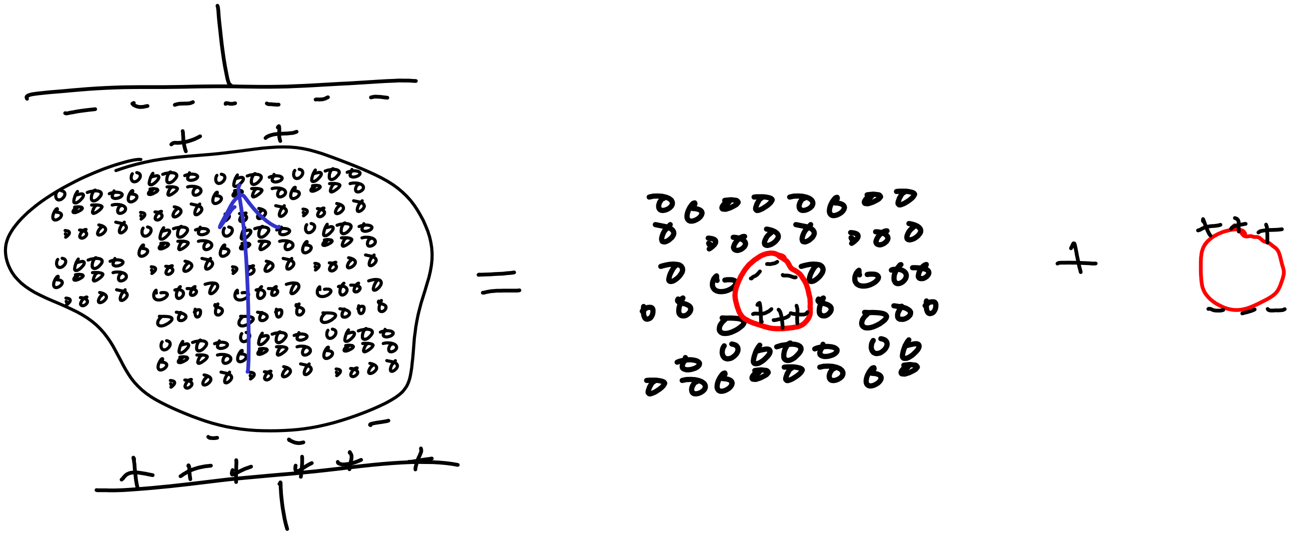
Homework problem 2.) At what frequency would the feedback signal go from complete negative to complete positive feedback if the only time delay is from the rudder back to the amplifier and is of magnitude tau?

The wavecrest is delayed by half a period to change from positive to negative feedback.

$$\text{Period} = T \qquad \frac{1}{2}T = \tau$$

$$\nu = \frac{1}{T} = \frac{1}{2\tau}$$

Homework problem 3. (a) Using the field of the hole to polarize an atom derive an expression for the dipole moment per unit volume that involves the atomic polarizability alpha. (b) Write an expression for the capacitance in terms of this atomic property.



$$E_{total} = E_{hole} + \underbrace{E_{plug}}_{= -\frac{P}{3\epsilon_0}}$$

$$E_{hole} = E_{total} - E_{plug} = E_{tot} + \frac{P}{3\epsilon_0}$$

The dipole moment of an atom is

$$p = \alpha E_{hole}$$

Big P for a chunk of matter is the dipole moment per atom times the number of atoms per volume.

$$P = N\alpha\left(E_{tot} + \frac{P}{3\epsilon_0}\right)$$

$$P = \underbrace{\frac{N\alpha}{1 - \frac{N\alpha}{3\epsilon_0}}}_{\epsilon_0 \chi_e} E_{tot} \Rightarrow \chi_e = \frac{N\alpha/\epsilon_0}{1 - \frac{N\alpha}{3\epsilon_0}}$$

microscopic property

Macroscopic property

Since the capacitance is $(1+\chi_e)C_0$

$\uparrow$ vacuum value

we can find χ_e experimentally

and determine α the atomic property of atoms

from a macroscopic measurement.

$$V_f = \frac{V_0}{1+\chi_e} = \frac{V_0}{K}$$

Homework problem 4. Using these partical derivative show

$$C_{final} = K C_0$$

$$Q_f = C_{final} V_f = Q_0$$

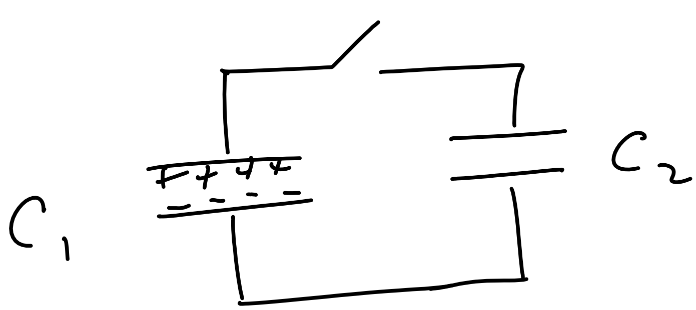
$$\delta V = \frac{\partial V}{\partial Q}\Big|_V \underset{0}{\underset{\parallel}{\delta Q}} + \frac{\partial V}{\partial C}\Big|_Q \delta C$$

$$\delta V = -\frac{Q_0}{C^2}\delta C \qquad \delta C = (K-1)C$$

$$\delta V \simeq -\frac{Q}{C^2}C(K-1) = -\frac{Q}{C}\left(\frac{\epsilon}{\epsilon_0}-1\right) = -\frac{Q}{C}\left(\frac{\epsilon_0(1+\chi_e)}{\epsilon_0}-1\right)$$

$$V_f - V_0 = -V_f\chi_e \qquad V_f = \frac{V_0}{1+\chi_e} = \frac{V_0}{K}$$

Homework problem 5. Capacitor 1 is charged to voltage V0 before it is connected to capacitor 2. What is the final charge on the second capacitor when the switch is closed?



Conservation of charge: $$Q_{initial} = Q_{final}$$

$$\parallel$$

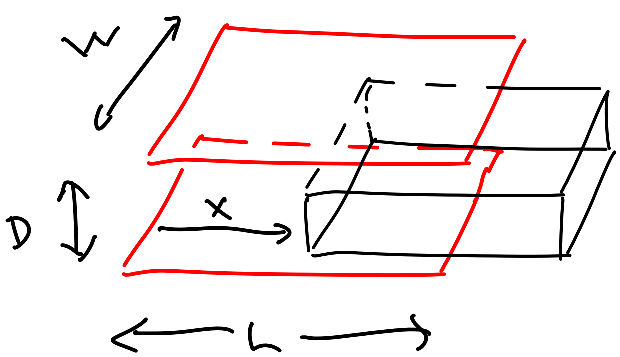
$$C_1 V_0 = Q_{1\,final} + Q_{2\,final}$$

Voltage is the same across both caps:

$$V_{fin} = Q_{1\,final}\, C_1 = Q_{2\,final}\, C_2$$

Two eqns in two unknows, the final charges.

Homework problem 6.) Derive an expression for the force on the slab using the geometry below.



$$F_{cap} = -\frac{dW_{me}}{dr} \rightarrow -\frac{dPE}{dx}$$

$$W_{me} = PE = \frac{1}{2}\frac{Q^2}{C(x)}$$

This is two caps in parallel.

$$C_{vac} = \epsilon_0 \frac{wx}{D} \qquad C_{glass} = K C_{vac} = K \epsilon_0 \frac{w(L-x)}{D}$$

$$C_{tot} = C_{vac} + C_{glass} = \frac{\epsilon_0}{D}\left(Wx + KWL - KWx\right)$$

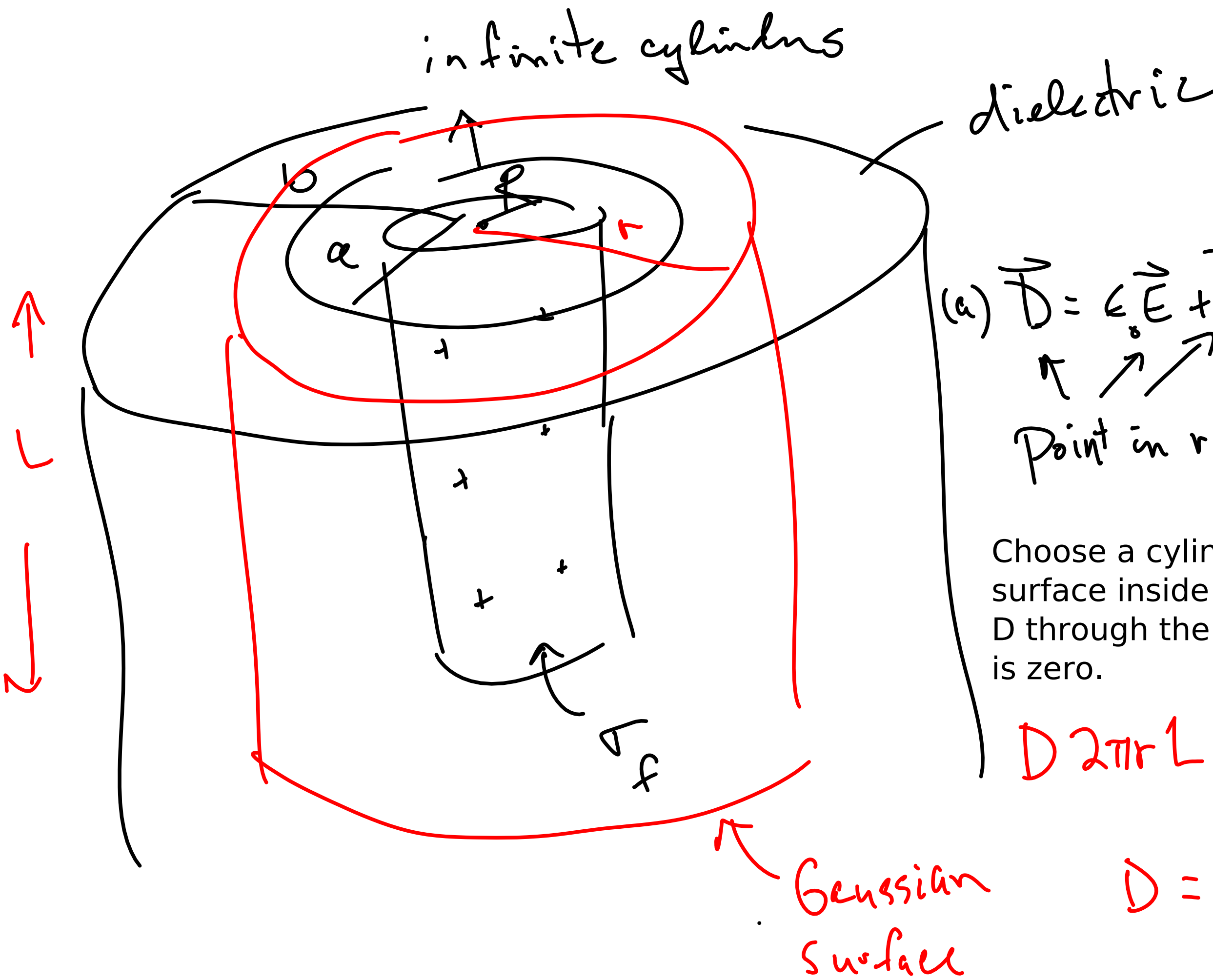
$$= \frac{\epsilon_0}{D}\left(Wx(1-K) + KWL\right) \qquad K = \frac{\epsilon}{\epsilon_0} = \frac{\epsilon_0(1+\chi_e)}{\epsilon_0}$$

$$C_{tot} = \frac{w\epsilon_0}{D}\left(x(1-1-\chi_e) + KL \right)$$

$$= \frac{w\epsilon_0}{D}\left(x\chi_e + KL \right)$$

$$W_{me} = -\frac{d}{dx}\left[\frac{1}{2}\,\frac{Q^2}{\frac{w\epsilon_0}{D}(x\chi_e + KL)} \right]$$

Homework problem 7: Use the integral form of Gauss's law to determine the electric field in the dielectric shown below. (b) find the surface bound charge on both surface of the linear material of suceptibility χ_e.



(a) $\vec{D} = \epsilon_0\vec{E} + \vec{P}$

Point in radial direction

Choose a cylindrical Gaussian surface inside the material. D through the endcaps is zero.

$$D\,2\pi r L = \sigma_f\,2\pi R L$$

$$D = \sigma_f \frac{R}{r}$$

$$D = \epsilon E = \sigma_f \frac{R}{r} \Rightarrow E = \frac{\sigma_f}{\epsilon}\frac{R}{r}$$

(b) $\sigma_b = \vec{P}\cdot\hat{n} = \epsilon_0\chi_e\vec{E}\cdot\hat{n} = \epsilon_0\chi_e\frac{\sigma_f}{\epsilon}\hat{r}\cdot\hat{n}\frac{R}{r}$

Inside surface $r=a$ & $\hat{r}\cdot\hat{n}=-1$ so

$$\sigma_b = -\frac{\epsilon_0}{\epsilon_0(1+\chi_e)}\sigma_f\frac{R}{a}$$

Outside surface $r=b$ & $\hat{r}\cdot\hat{n}=1$ so

$$\sigma_b = \frac{1}{1+\chi_e}\sigma_f\frac{R}{b}$$

$$Q_{\text{on material}} = \sigma_b^{\text{inner}}2\pi aL + \sigma_b^{\text{outer}}2\pi bL$$

$$= \frac{-1}{1+\chi_e}\sigma_f R2\pi L + \frac{1}{1+\chi_e}\sigma_f R2\pi L$$

$$= 0$$

as expected since there is

no $\rho_b = -\vec{\nabla}\cdot\vec{P} = -\vec{\nabla}\cdot\epsilon_0\chi_e\vec{E} = -\vec{\nabla}\cdot\epsilon_0\chi_e\frac{\sigma_f}{\epsilon_0}\frac{R}{r}\hat{r}$

in cylindrical coords:

$$\vec{\nabla}\cdot\frac{1}{r}\hat{r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{1}{r}\right) + 0 + 0 = 0$$