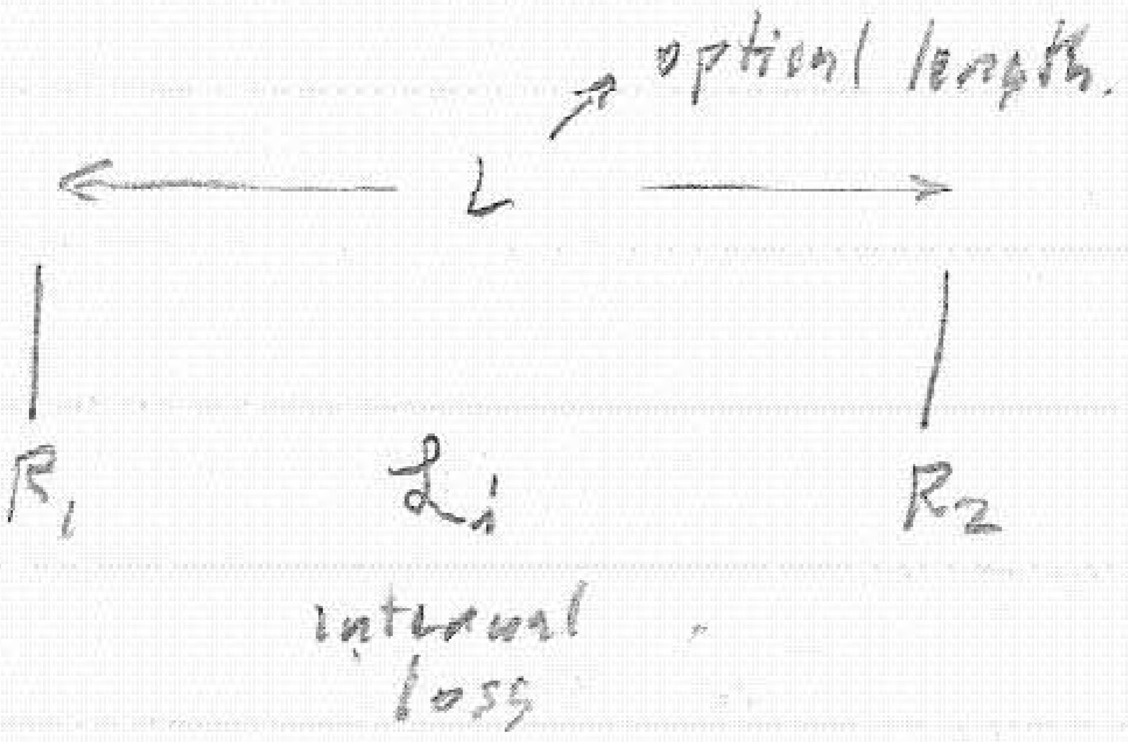
Rate equations for oscillators

review

<u>passive cavity</u>



start with intensity I_0

after m passes (round trips)

$$I(t_m) = \left[R_1 R_2 (1-T_i)^2\right]^m I_0 \qquad t_m = \frac{2L}{c}$$

work with ϕ = number of photons in cavity $\propto \frac{I \cdot A}{h\nu}$

$$\phi(t_m) = \left[R_1 R_2 (1-T_i)^2\right]^m \phi_0$$

the photon number is exponentially damped in time.

(assuming low loss/r.t.)

$$\phi(t_m) = \phi_0 e^{-t_m/\tau_c} = \phi_0 e^{-2mL/c\tau_c}$$

$\therefore$ photon lifetime $\tau_c = -\frac{2L}{c} \frac{1}{\ln\left[R_1 R_2 (1-\mathcal{L}_i)^2\right]}$

$$\equiv \frac{L}{c\gamma}$$

γ = "logarithmic cavity loss" per <u>pass</u> ($\frac{1}{2}$r.t.)

$$= -\frac{\ln\left[R_1 R_2 (1-\mathcal{L}_i)^2\right]}{2}$$

so passive cavity follows eqn

$$\frac{d\phi}{dt} = -\frac{\phi}{\tau_c}$$

<u>gain</u>

within the material $\quad \frac{dF}{dz} = \sigma_{21} F \underbrace{\left[N_2 - \frac{g_2}{g_1} N_1\right]}_{\text{inversion density}}$

F = photon flux.

assuming everything is spatially uniform (F, N_1, N_2)

then $\left[N_2 - \frac{g_2}{g_1} N_1\right] h\nu$ = stored energy density = U_{stored}

and $h\nu/\sigma_{21}$ = saturation fluence = F_{sat}

$$\rightarrow F(z) = F_0 e^{gz} \qquad g = \frac{U_{stored}}{F_{sat}} = \text{gain coeff.}$$

if medium has a length l

$$F_1 = F_0 e^{gl} = F_0 \underbrace{G_0}_{\text{small signal gain (one pass)}}$$

in terms of photon number

$$\phi(t_m) = \phi_0 G_0^{2m} = \phi_0 e^{t_m/\tau_g} = \phi_0 e^{\frac{2mL}{c\tau_g}}$$

again assume exponential behavior

τ_g ~ photon gain timescale

$$e^{2mL/c\tau_g} = G_0^{2m} = e^{2mgl}$$

$$\frac{L}{c\tau_g} = gl = \left(N_2 - \frac{g_2}{g_1} N_1\right) \sigma_{21} l$$

stim em. rate: $\frac{1}{\tau_g} = \left(N_2 - \frac{g_2}{g_1} N_1\right) c \sigma_{21} \frac{l}{L}$

define stimulated emission coeff / photon

$$B = \sigma_{21} c \frac{l}{L} \cdot \frac{1}{V_a} \rightarrow \text{mode volume}$$

$$\rightarrow \frac{d\phi}{dt} = + \frac{\phi}{\tau_g} = \phi \left(N_2 - \frac{g_2}{g_1} N_1\right) V_a B$$

combine gain and loss:

$$\frac{d\phi}{dt} = \underbrace{\left(N_2 - \frac{g_2}{g_1} N_1\right)}_{= N_2 \text{ for ideal 4-level system}} V_a B \phi - \frac{1}{\tau_c} \phi$$

Also keep track of population $N_2(t)$

$$\frac{dN_2}{dt} = \underset{\downarrow \atop \text{pump}}{R_p} - \underbrace{B\phi N_2}_{\text{stimulated}} - \underset{\rightarrow \text{ spontaneous}}{\frac{N_2}{\tau}}$$

Notes:

These equations are simple, but –

- assume spatial uniform. this eqn set gives good baseline before a more complicated / complete analysis.
- assume one spatial, longitudinal mode. Later, acct. for phase.
- assume ∞ rates A_{32}, A_{10}
  - > $R_p(t)$ ok, but require $A_{32}, A_{10} \gg R_p$
- R_p is an effective value, averaged over volume of mode.
- spatial variation in N_2, ϕ (e.g. standing wave → spatial hole burning)
- inhomogeneous broadening can lead to interaction w/ subset of population.

Losses: some portion → output coupling.

let m_2 → output coupler (OC)

$$\frac{1}{\tau_c} = \frac{\gamma_i c}{L} + \frac{\gamma_1 c}{2L} + \frac{\gamma_2 c}{2L} \qquad \gamma\text{'s} = \text{log. loss.}$$

OC transmission $= 1 - R_2$ assuming no mirror loss.

output power $= (1 - R_2)$ intracavity power

$$P_{out} = \frac{\gamma_2 c}{2L} h\nu\, \phi$$

steady state laser behavior: $d\phi/dt = \frac{dN_2}{dt} = 0$

transients: $R_p(t)$, solve coupled eqns. → q-switching, relaxation osc. etc.

Now get a critical $R_p \Rightarrow R_{cp} = \frac{N_c}{\tau}$ (neglect stim. emission)
pump beats out fluorescence.

$$R_{cp} = \frac{\gamma}{\sigma l \tau}$$

with $R_p > R_{cp}$ $\phi \uparrow$ until some stable level.

set $\frac{dN}{dt} = \frac{d\phi}{dt} = 0$

$$\frac{dN}{dt} = 0 = R_p - B\phi_0 N_0 - N_0/\tau$$

$$\frac{d\phi}{dt} = 0 = \underbrace{\left(BV_a N_0 - \frac{1}{\tau_c}\right)}\phi_0 \rightarrow \phi_0 = \frac{R_p - N_0/\tau}{BN_0}$$

$$N_0 = \frac{1}{\tau_c B V_a} \longrightarrow = R_p \tau_c V_a - \frac{1}{B\tau}$$

$$= N_c \qquad \text{or} \quad \phi_0 = \tau_c V_a (R_p - N_0/\tau)$$

$\therefore$ always maintain critical inversion density even for $R_p > R_{pc}$

extra pump power $\rightarrow$ output

$V_a R_p$ = # atoms pumped / time

$$\phi_0 = \underbrace{\tau_c \cdot V_a R_p}_{\substack{\text{cavity storage time} \\ \times \text{ \#atoms pumped/time}}} - \underbrace{(N_0/\tau)\tau_c V_a}_{\text{fluorescence losses.}}$$

alt form

$$\phi_0 = \frac{\tau_c}{\tau} V_a N_0 \left(\frac{\tau R_p}{N_0} - 1\right) = V_a N_0 \frac{\tau_c}{\tau}\left(\frac{R_p}{R_{cp}} - 1\right)$$

$V_a N_0 \downarrow \frac{V_a}{l}\frac{\gamma}{\sigma}$, $\frac{V_a}{l} \downarrow$ cross-sect. area of mode.

$\frac{R_p}{R_{cp}} \downarrow \frac{P_{in}}{P_{th}}$

out put power:

$$P_{out} = \frac{\gamma_2 c}{2L} h\nu \Phi_0 = h\nu \frac{\gamma_2 c}{2L} \frac{A_b \gamma}{\sigma} \frac{\gamma_c}{\tau} \left(\frac{P_p}{P_{th}} - 1 \right)$$

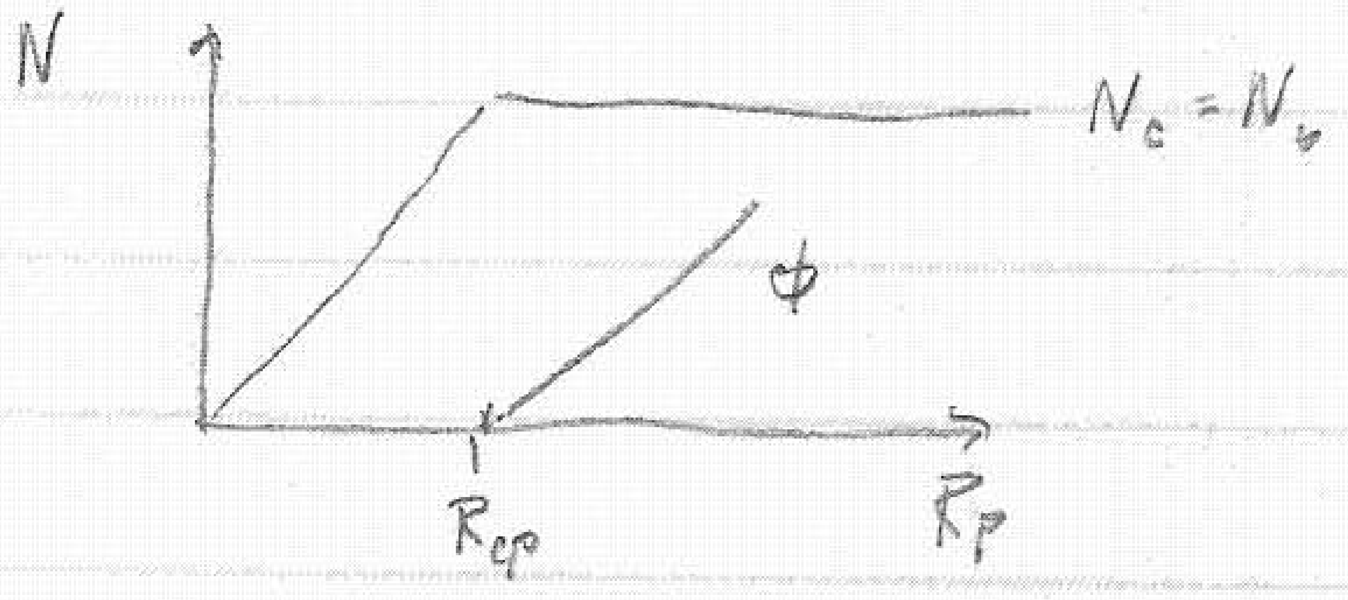
$$\frac{h\nu}{\sigma\tau} = I_s$$

$$P_{out} = A_b I_s \left(\frac{\gamma_2}{\tau} \right) \left(\frac{P_p}{P_{th}} - 1 \right)$$

mode area

slope eff. $\eta_s = \frac{d P_{out}}{d P_p} = A_b I_{sat} \frac{\gamma_2}{\tau} \frac{1}{P_{th}}$

measure P_{th}, η_s A_b → cavity losses.



pump threshold (uniform pumping):

threshold excitation rate $R_{cp} = \frac{\gamma}{\sigma l} \frac{1}{\tau}$

losses

→ fluoresc.

effective vol.

connect to pump power:

$$R_p = \eta_p \frac{P}{A l \, h\nu_{mp}}$$

photon arrival rate

pump eff.

vol. of gain

$$\eta_p \frac{P_{th}}{A l \, h\nu_{mp}} = \frac{\gamma}{\sigma l} \frac{1}{\tau}$$

$$P_{th} = \frac{\gamma}{\eta_p} \frac{A}{\sigma} \frac{h\nu_{mp}}{\tau} = \frac{\gamma}{\eta_p} A \cdot I_{sat} \cdot \frac{h\nu_{mp}}{h\nu_0}$$