Effect of phase mismatch

- assume $\Delta k = 0 \rightarrow A_3(z) \propto z \qquad I_3(z) \propto z^2$
- for $\Delta k \neq 0$:

$$\frac{dA_3}{dz} = s e^{i\Delta k z} \qquad s = \frac{2 d_{eff}\, i\omega_3}{n_3 c} A_1 A_2$$

integrate directly:

$$A_3(z) = s\int_0^L e^{i\Delta k z}\, dz = s\left(\frac{e^{i\Delta k L} - 1}{i\Delta k}\right)$$

$$= s e^{i\Delta k L/2} \frac{\left(e^{i\Delta k L/2} - e^{-i\Delta k L/2}\right) L}{2i\Delta k L/2}$$

$$= L s e^{i\Delta k L/2} \frac{\sin(\Delta k L/2)}{\Delta k L/2} = L s e^{i\Delta k L/2}\, \underline{\mathrm{sinc}\left(\frac{\Delta k L}{2}\right)}$$

Intensity is $\propto |A_3|^2$

$$I_i = \frac{n_i c}{8\pi}|\mathcal{E}|^2 \boxed{= \tfrac{1}{2} n_i \epsilon_0 c |\mathcal{E}|^2} \qquad E = \mathcal{E}\cos(kz - \omega t) = A e^{i(kz-\omega t)} + c.c.$$

$$= \frac{n_i c}{2\pi}|A|^2 \boxed{= n_i \epsilon_0 c |A|^2} \qquad A = \mathcal{E}/2$$

$$\rightarrow I_3(L) = \frac{512\pi^5 d_{eff}^2 I_1 I_2}{n_1 n_2 n_3 \lambda_3^2 c} L^2\, \mathrm{sinc}^2(\Delta k L/2) \qquad \boxed{\frac{32\pi^2}{\epsilon_0}} \text{ SI}$$

notes:
- $I_3 \propto d_{eff}^2$ magn. of deff is impt.
- I_3 is linear in I_1, I_2
- for $\Delta k \neq 0$, $I_3 \propto \frac{\sin^2(\Delta k L/2)}{(\Delta k)^2}$

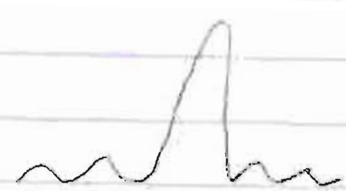
power osc. along length.

- as $\Delta k \to 0$ period $\uparrow$

ampl. $\uparrow$

- $\Delta k \neq 0$ fixed L vary Δk

$I_3 \propto \mathrm{sinc}(\Delta k L/2)$



Anisotropic media: birefringence.

in an isotropic medium:

$$\vec{D} = \varepsilon \vec{E} \qquad\qquad \vec{D} = \varepsilon_0 \varepsilon \vec{E}$$

if linear, ε doesn't depend on $\vec{E}$

$\vec{D} \parallel \vec{E}$

in an anisotropic medium, $\vec{D} \nparallel \vec{E}$

$$\underset{\text{applied}}{\vec{D} = \vec{E}} + \underset{\text{induced polarization}}{4\pi \vec{P}} \qquad\qquad \boxed{\vec{D} = \varepsilon_0 \vec{E} + \vec{P}}$$

b/c of asymmetry of crystal, $\vec{P} \nparallel \vec{E}$ e.g. 

write $\vec{D} = \overleftrightarrow{\varepsilon} \cdot \vec{E}$

$$\overleftrightarrow{\varepsilon} = \text{dielectric tensor} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix}$$

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if material is not lossy, energy considerations $\rightarrow$ $\overleftrightarrow{\varepsilon}$ is Hermitian $\varepsilon_{ij} = \varepsilon_{ji}^*$

∴ We can choose the orientation of coordinates to diagonalize (in most crystals)

$$\rightarrow \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} \varepsilon_x & & \\ & \varepsilon_y & \\ & & \varepsilon_z \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

this coord. system is aligned along the crystal axes.

so if $\vec{E} = E_x(x,y,z)\,\hat{x}$

$\vec{D} = D_x \hat{x} = \varepsilon_x E_x$ only.

biaxial crystal: $\varepsilon_x \neq \varepsilon_y \neq \varepsilon_z$

uniaxial crystal: one pair is the same. e.g. $\varepsilon_x = \varepsilon_y \neq \varepsilon_z$

isotropic crystal $\varepsilon_x = \varepsilon_y = \varepsilon_z = \varepsilon$ $\quad \vec{D} = \varepsilon \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \vec{E}$

"dichroic" absorption depends on polarization e.g. polaroid.