MATH-300 - Foundations of Adv. Mathematics **Field 2009**
Midterm 2 **Due: Thursday June 11, 3:00pm**

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1. Let Λ be a nonempty indexing set and let $\mathcal{A} = \{A_\alpha \,|\, \alpha \in \Lambda\}$ be an indexed family of sets. Also, assume that $\Gamma \subseteq \Lambda$ and that $\Gamma \neq \varnothing$. Prove:
$$\bigcup_{\alpha \in \Gamma} A_\alpha \subseteq \bigcup_{\alpha \in \Lambda} A_\alpha$$

2. Using mathematical induction show that given any two real numbers a and b, $a - b$ is a factor of $a^n - b^n$ for all $n \in \mathbb{N}$.

3. Prove
$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} \geq \sqrt{n} \text{ for all } n \in \mathbb{N}.$$

4. Let $f : S \to T$ be a function with C and D subsets of T. Prove: $f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$.

5. Let $f : S \to T$ be a function. Prove that $f(A \cap B) = f(A) \cap f(B)$ for all subsets A and B of S if and only if f is an injection.