Reading

12.1.1 – 12.1.3

pgs. 477–499

Einstein Summation Convention

Sum over repeated indices

$x=1$, $y=2$, $z=3$

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$= A_1 B_1 + A_2 B_2 + A_3 B_3$$

$$= \sum_{i=1}^{3} A_i B_i$$

$$= A_i B_i$$

$$\vec{\nabla}\cdot\vec{E} = \partial_i E_i$$

Levi-Civita T.A.T.

$\varepsilon_{ijk} = +1$ if i,j,k is an even permutation of $1,2,3$

$= -1$ if i,j,k is an odd permutation of $1,2,3$

$= 0$ if any indices are the same

$$(\vec{A}\times\vec{B})_i = \varepsilon_{ijk} A_j B_k$$

$$(\vec{A}\times\vec{B})_3 = \varepsilon_{3jk} A_j B_k = A_1 B_2 - A_2 B_1$$

$$A_x B_y - A_y B_x$$

$$\varepsilon_{ijk}\varepsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \varepsilon_{ijk} A_j \varepsilon_{klm} B_l C_m$$

$$= \varepsilon_{ijk}\varepsilon_{klm} A_j B_l C_m$$

$$= \varepsilon_{ijk}\varepsilon_{lmk} A_j B_l C_m$$

$$= (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) A_j B_l C_m$$

$$= B_i A_m C_m - C_i A_l B_l$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

$$\vec{A}\times(\vec{\nabla}\times\vec{B}) + \vec{B}\times(\vec{\nabla}\times\vec{A}) + (\vec{A}\cdot\vec{\nabla})\vec{B} + (\vec{B}\cdot\vec{\nabla})\vec{A}$$

$$\varepsilon_{ijk} A_j \varepsilon_{klm} \partial_l B_m + \varepsilon_{ijk} B_j \varepsilon_{klm} \partial_l A_m + A_j \partial_j B_i + B_j \partial_j A_i$$

$$(\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) A_j \partial_l B_m + (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) B_j \partial_l A_m$$

$$A_m \partial_i B_m - A_l \partial_l B_i + B_m \partial_i A_m - B_l \partial_l A_i + A_j \partial_j B_i + B_j \partial_j A_i + A_j \partial_j B_i + B_j \partial_j A_i$$

$$\partial_i (A_m B_m) = \vec{\nabla}(\vec{A}\cdot\vec{B})$$

$$A_m \partial_i B_m + B_m \partial_i A_m$$