MATH348 - February 13, 2009	**NAME**:
Exam I - 50 Points - 50 minutes	**SECTION**:

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

1. (10 Points) True/False and Short Response

   (a) Mark each statement True or False.

   i. The columns of a matrix $\mathbf{A}$ are linearly independent if the equation $\mathbf{Ax} = \mathbf{0}$ has only the trivial solution, $\mathbf{x} = \mathbf{0}$.

   ii. If the equation $\mathbf{Ax} = \mathbf{0}$ has a nontrivial solution, then $\mathbf{A}$ has fewer than n pivot positions.

   iii. If the columns of $\mathbf{A}$ are linearly dependent, then $\det(\mathbf{A}) = 0$.

   iv. The columns of an $n \times n$ invertible matrix form a basis for $\mathbb{R}^n$.

   v. The column space of $\mathbf{A}$ is the set of all solutions to $\mathbf{Ax} = \mathbf{b}$.

   (b) Please respond to <u>**one** of the following</u>:

   i. Suppose $\mathbf{A}$ is a 4×3 matrix and $\mathbf{b}$ is a vector in $\mathbb{R}^4$ with the property that $\mathbf{Ax} = \mathbf{b}$ has a unique solution. What can you say about the reduced echelon form of $\mathbf{A}$?

   ii. Suppose $\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ what is the determinant of $\mathbf{A}$? List three more equivalent properties/characterizations of $\mathbf{A}$.

   iii. Suppose that $\mathbf{Ax} = \mathbf{0}$ and $\mathbf{x} \neq \mathbf{0}$ is an eigenvector. What does the eigenvalue corresponding to this $\mathbf{x}$ have to be? Explain.

2. (10 Points) Given the following matrix $\mathbf{A}$ and its associated eigenvectors $\mathbf{y}_1$ and $\mathbf{y}_2$:

$$\mathbf{A} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad \mathbf{y}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \mathbf{y}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{b}_3 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}.$$

(a) Find the solution to $\mathbf{Ax} = \mathbf{b}_1$.

(b) Find the solution to $\mathbf{Ax} = \mathbf{b}_2$.

(c) Find the solution to $\mathbf{Ax} = \mathbf{b}_3$.

(d) Find $\mathbf{A}^4$

3. (10 Points) Given,

$$\begin{aligned} x_1 + hx_2 &= 2 \\ 4x_1 + 8x_2 &= k. \end{aligned}$$

Choose h and k such that the system has:

(a) No Solution

(b) A Unique Solution

(c) Many Solutions

4. (10 Points) Given,

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix}$$

Find a basis and the dimension of the null-space and column-space of $\mathbf{A}$.

5. (10 Points) Given,

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

Find all eigenvalues and eigenvectors of $\mathbf{A}$.